Module 7: Introduction to Transportation Systems

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CE 4240/5240 — Intro to Infrastructure Systems Engineering

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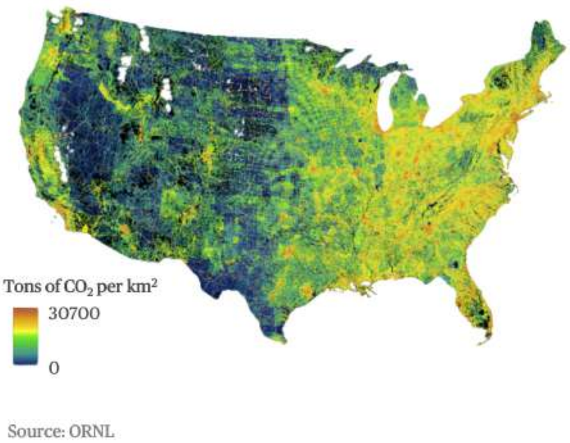
Overview

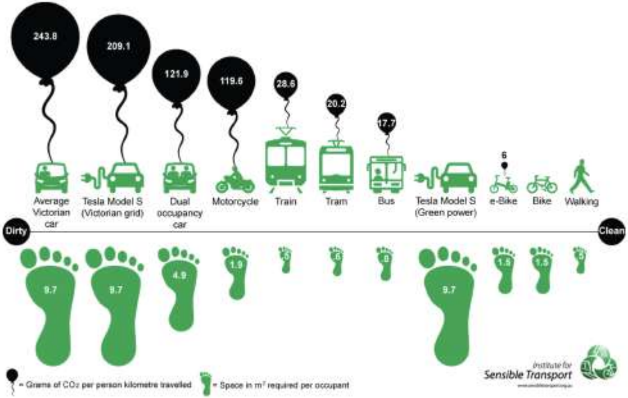
Emission: elephant in the room

Highest-emitting sector in the United States? Transportation!

- Transportation is the largest source of greenhouse gas emissions in the United States, and CO_2 emissions represent roughly 97% of the global warming potential of all greenhouse gas emissions from transportation.



Emission: space and emissions by traffic modes



Source: Institute for Sensible Transport

Emission: space and emissions by traffic modes

On-road transportation emissions (CO_2)

Geography	Total emissions per year, tons	Average by area, tons/mi²/year	Average by population, tons/per/resident	% difference from National average
United States	1,747,657,704	460	5.38	–
Southeast Region	378,679,522	959	5.76	7.43%
Tennessee	41,989,164	996	6.25	16.17%
Davidson County	6,026,327	11,566	8.76	62.82%

Table 1. Differences in Transportation emissions (measured as the tons of CO_2 per resident emitted on-road in 2017) at the county, state, regional, and national levels. Davidson County residents emit 62.82% more than the national average, whereas the state of TN and the SE region emit 16.17% and 7.43% more than the national average.

Data sources: ORNL DARTEv2 (Database of Road Transportation Emissions), FHWA HPMS, Census

Created by © Junyi Ji

Emission: action is needed, now!

November 22, 2023
Biden-Harris Administration Finalizes Greenhouse Gas Emissions Reduction Tool, Moves Climate Change Performance Measure Forward

- Carbon Reduction Program (6.4 billion)
- National Electric Vehicle Infrastructure (NEVI) formula program (5 billion)
- Charging and Fueling Infrastructure (CFI) discretionary grant (2.5 billion)
- Congestion Relief Program (250 million)
- Reduction of Truck Emissions at Port Facilities Program (400 million)
- Low or No Emission Vehicle Program (5 billion)
- Transportation Alternatives Set-Aside program (7.2 billion)
- Transit Oriented Development (TOD) Program (69 million)

Safety: how far are we from Vision Zero?

Vision Zero: a multi-national road traffic safety project that aims to achieve a roadway system with **NO** fatalities or serious injuries involving road traffic. It started in Sweden and was approved by their parliament in October 1997.

Figure: Traffic Safety in Nashville Today

However, each year, more than 42,000 people — the population of a small city — are needlessly killed on American streets and thousands more are injured.

Congestion: the curse of transportation

Braess's paradox: the observation that adding one or more roads to a road network can slow down overall traffic flow through it.

Figure: If the solution to urban traffic is to provide more capacity to cars... Case in Alexandria, Egypt

Congestion: can more lanes solve the problem?

Figure: Burning Man festival 2023 exit time 5 hours for 5 mile

Congestion: everywhere and everyday

Stop-and-go traffic (phantom jam): traffic that unexpectedly stops and then begins moving again without a clear cause is a major source of annoyance for drivers. Jams can develop spontaneously on roads that are at full capacity with free-moving traffic, often initiated by just one driver reducing speed.

Figure: Mathematical Society of Traffic Flow, shockwave traffic jam recreated for first time, https://youtu.be/7wm-pZp_mi0?si=7PJt86Gc0rbHkAL-

Congestion: simulation tools

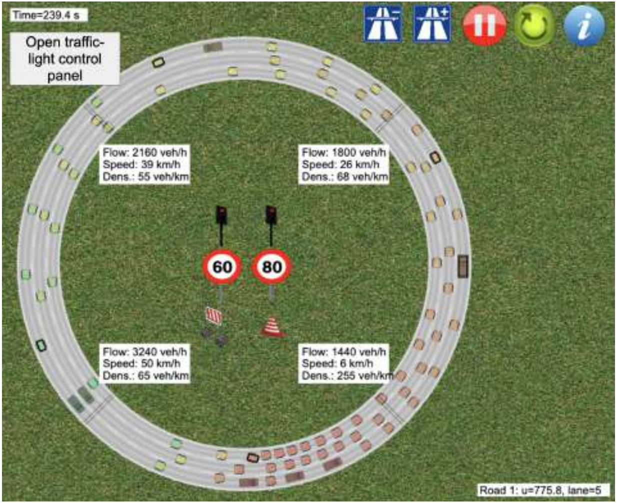


Figure: Interactive simulation tool, developed by Martin Treiber from TU Dresden, https://traffic-simulation.de/ring.html

Congestion: concept of space-time diagram

Space and time hold critical importance in the realm of transportation.

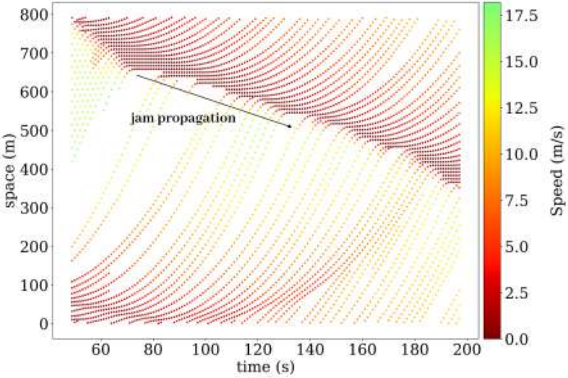


Figure: A space-time diagram, derived from simulation data, visually represents the movement of vehicles over time and space. In this diagram, each point indicates a vehicle's position at a given time. The color coding of these points reflects their speed: red signifies congestion, while green indicates smooth traffic flow. This makes it easier to track and understand traffic dynamics. Code available here: https://github.com/junyi9/CE5240/tree/main/Exp1_space_time_diagram

Congestion: real-world testbed could tell you more...

The Tennessee Department of Transportation's I-24 Mobility Technology Interstate Observation Network (MOTION) is a four-mile section of I-24 in the Nashville-Davidson County Metropolitan area with 294 ultra-high definition cameras. Those images are converted into a digital model of how every vehicle behaves with unparalleled detail.

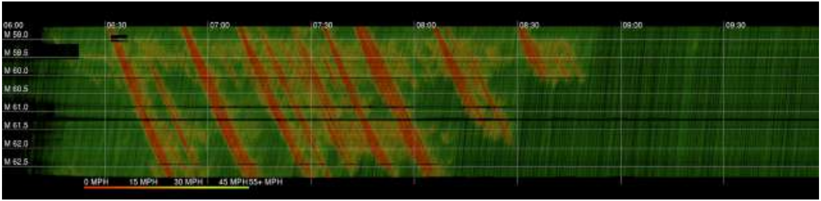


Figure: I-24 MOTION testbed (`i24motion.org`), contributed by Vanderbilt University, captured world's largest space-time diagram (figure created by Gergely Zachár and Derek Gloudemans), 4 hours and 4 miles

Empirical Facts:

- Wave propagates nearly at the same speed (about -12.5 mph).
- Congestion jam happens everyday and seems unavoidable...

How can we solve the challenges mentioned above?

- So what is the nature of transportation and mobility?
- All the challenges here are within the scope of dynamical systems - human-involved, complex, highly nonlinear and large-scale.
- Management, decisions and actions are also everywhere in the world's of transportation,
- What do we need for the transportation system as the basics for all the potential solutions to challenges? Model!

Traffic models

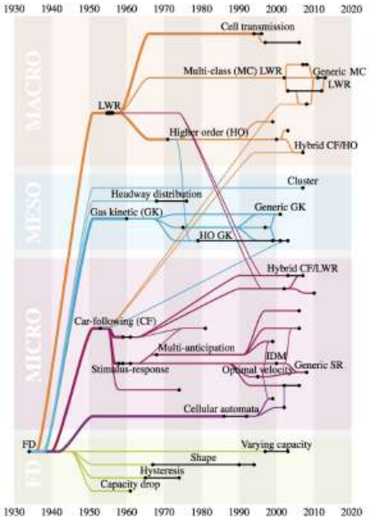


Figure: History of traffic models, from TRB report E-C195

Microscopic models: individual dynamics

Microscopic models : model individual vehicle behavior and vehicle-to-vehicle interaction.
Continuous in space and time: car-following model can be described as coupled ordinary differential equations (ODEs) as in Newtonian dynamics:

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = v_i, \quad \frac{\mathrm{d}v_i}{\mathrm{d}t} = f_i\left(x_i, x_{i-1}, v_i, v_{i-1}, ..\right) \tag{1}$$

where x_i are the location of vehicle i, $f_i(\cdot)$ is the car-following model.

$$\frac{\mathrm{d}v_i}{\mathrm{d}t} = f_i(v_i, s_i, \Delta v_i) \tag{2}$$

where $s_i = x_{i-1} - x_i - l_{i-1}$, $\Delta v = v_{i-1} - v_i$, and l_{i-1} is the length of the vehicle $i - 1$.
Quiz: state-space representation of a car-following model?

Microscopic models: a general state-space representation

1. Selection of the state vector

$$\mathbf{x}_i = \begin{bmatrix} \Delta x_i \\ v_i \end{bmatrix} \tag{3}$$

2. Delay-free car-following models:

$$\Delta \dot{x}_i = v_{i-1} - v_i \tag{4}$$

$$\dot{v}_i = f_i(v_i, \Delta x_i, v_{i-1} - v_i) = f_i(v_i, \Delta x_i, \Delta \dot{x}_i) \tag{5}$$

3. If the car-following model is a linear model:

$$\Delta \dot{x}_i = v_{i-1} - v_i \tag{6}$$

$$\dot{v}_i = \alpha_1 v_i + \alpha_2 \Delta x_i + \alpha_3 \Delta \dot{x}_i \tag{7}$$

Homework...

Macroscopic models: aggregated measurements

Aggregated measurements/quantities in macroscopic models:

- Flow: the traffic flow is defined as the number of vehicles ΔN passing the cross-section at location x within a time interval Δt

$$Q(x,t) = \frac{\Delta N}{\Delta t} \tag{8}$$

- Speed: arithmetic mean speed and harmonic mean speed

$$V(x,t) = \langle v_\alpha \rangle = \frac{1}{\Delta N} \sum_{\alpha=\alpha_0}^{\alpha_0+\Delta N-1} v_\alpha \tag{9}$$

$$V_{\mathrm{H}}(x,t) = \frac{1}{\left\langle \frac{1}{v_\alpha} \right\rangle} = \frac{\Delta N}{\sum_{\alpha=\alpha_0}^{\alpha_0+\Delta N-1} \frac{1}{v_\alpha}} \tag{10}$$

- Density: the traffic density is defined as the amount of traffic per unit of road length (hard to be directly measured, why?)

$$\rho(x,t) = \frac{Q(x,t)}{V(x,t)} = \frac{\text{flow}}{\text{speed}} \tag{11}$$

Macroscopic models: aggregated dynamics

Fundamental diagram: relationship between speed, flow and density.

Figure: Greenshields measurement set up in the 1930s.

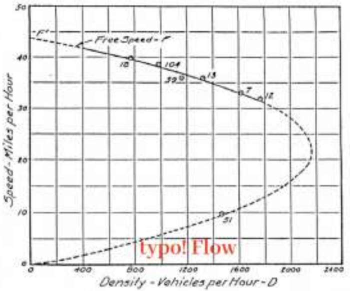


Figure: the first fundamental diagram captured by Greenshields.

Further research reveals various shapes of the fundamental diagram, yet the pattern remains consistent: traffic flow begins to decrease when it exceeds the critical density.

Macroscopic models: water is also a fluid system

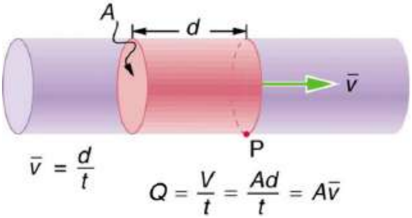


Figure: Flow rate is the volume of fluid per unit time flowing past a point through the area A. Here the shaded cylinder of fluid flows past point P in a uniform pipe in time t.

"Hyperbolic" – the nature of human traffic flow!

Urban traffic: definition

Urban traffic, characterized by intersections (with traffic signals), access points, and pedestrian activity, is more complex. It is typically modeled as a network system, often employing queuing theory for its dynamics.

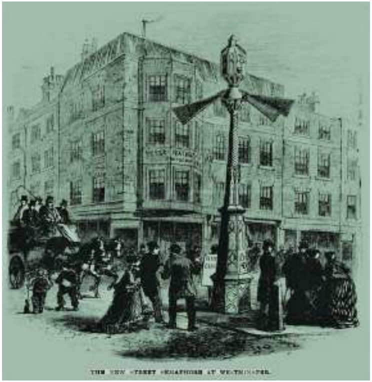


Figure: First signal control in the world, January 16, 1869 installed in Westminster, London

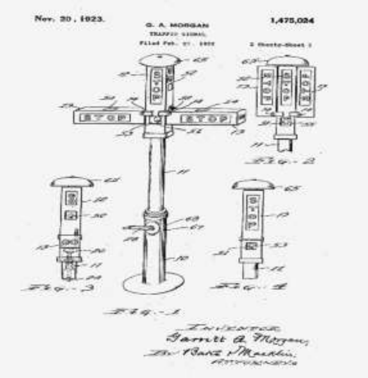


Figure: Garrett A. Morgan invented the first three-position traffic signal, Nov. 20, 1923 [**100 years**].

Urban traffic: framework

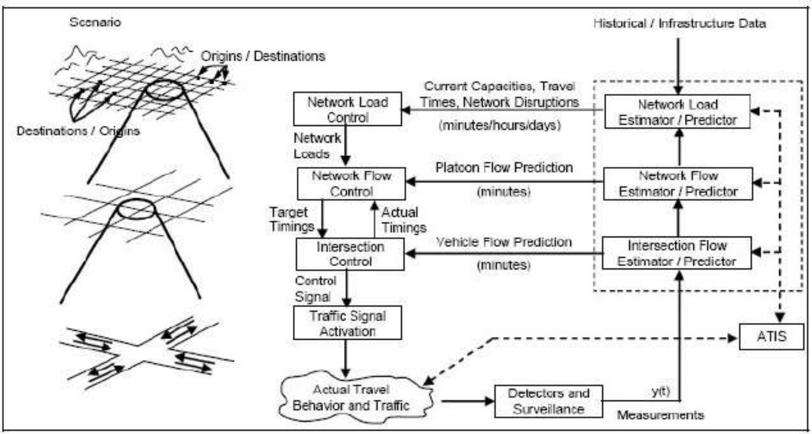


Figure: General Framework of urban traffic system (from FHWA Traffic Control Systems Handbook): elements include OD matrix (network level), vehicle routing (individual behavior on network), signal timing (intersection level)

Urban traffic: cases

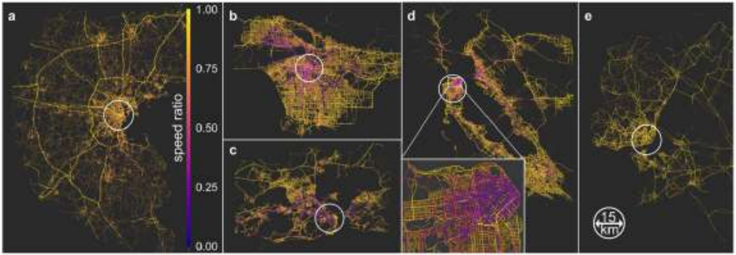


Fig. 2 Maps of the traffic networks for each city. The networks depicted represent the morning traffic conditions (at 09:00) on a weekday in **a** Boston, **b** Los Angeles, **c** Rio de Janeiro, **d** San Francisco Bay Area, and **e** Lisbon. The speed ratios are obtained by dividing traffic speed by the speed limit.

Figure: Understand the the collapse of urban traffic from a network perspective

Freeway traffic: definition

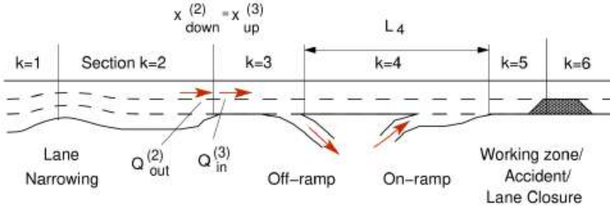
A freeway is a highway where access to the roadway is controlled. Drivers can only enter a controlled-access highway by ramps. Regarding the model, freeway traffic can be viewed as a flowing, continuous fluid.

Trivia: all freeways are highways, but not all highways are freeways.

General: concepts

Conservation Law in macroscopic traffic flow for a homogeneous road section.

$$\frac{\partial \rho}{\partial t} + \frac{\partial Q}{\partial x} = \frac{\partial \rho}{\partial t} + \frac{\partial (\rho V)}{\partial x} = 0$$



Useful resources are listed below:

- Martin Treiber, Traffic Dynamics and Simulation Class, link https://www.mtreiber.de/Vkmod/index.html
- Traffic flow on networks, Mauro Garavello, Benedetto Piccoli, 2006.

General: perspective from PDE

- We have three main macroscopic quantities: density ρ, flow Q, and local speed V.
- There is always the static hydrodynamic relation between these quantities arising directly by the definitions of ρ, Q, and V :

$$Q = \rho V \tag{12}$$

Conservation Law in macroscopic traffic flow for a homogeneous road section.

$$\frac{\partial \rho}{\partial t} + \frac{\partial Q}{\partial x} = \frac{\partial \rho}{\partial t} + \frac{\partial (\rho V)}{\partial x} = 0 \tag{13}$$

Two model-independent relations between the three quantities are always there. To make a macroscopic flow model that can be simulated, we need a third equation.

LWR: first-order model

Here, understanding the "first-order" is important.

- The third equation is $Q = Q_e(\rho)$, i.e. the relationship between density and flow (fundamental diagram), it assumes the **static (fixed)** relationship between density and flow
- Add the fundamental diagram to the equation.

$$Q(x,t) = Q_e(\rho(x,t))$$

  Q_e indicates that fundamental diagram is the equilibrium description of traffic flow.

- Conservation Law in LWR model

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(Q_e(\rho)) = \frac{\partial \rho}{\partial t} + Q_e'(\rho)\frac{\partial \rho}{\partial x} = 0 \quad \text{LWR Model}}$$

- Quiz: show $Q_e'(\rho) =$?
- Revise: the first fundamental diagram?

Greenshields: quadratic fundamental diagram

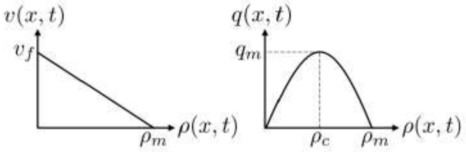


Figure: Greenshield's fundamental diagram: (left) Traffic speed versus traffic density. (right) Traffic flow versus traffic density.

$$v = v_f \left(1 - \frac{\rho}{\rho_m}\right)$$

$$Q(\rho) = v_f \rho \left(1 - \frac{\rho}{\rho_m}\right) = -\frac{v_f}{\rho_m}\rho^2 + v_f \rho$$

$$\rho_c = \frac{1}{2}\rho_m$$

Greenshields: discretization

- Godunov's scheme to transform the first-order PDE to the following ODE.

$$\dot{\rho}(x,t) = \frac{\partial \rho(x,t)}{\partial t} = -\frac{\partial Q(x,t)}{\partial x} \approx \frac{q(x,t) - q(x+l,t)}{l} \tag{14}$$

- Junction model equations in Greenshields (switch system):
  - Demand $(\delta(x,t))$ of a segment is the amount of traffic that wants to leave the segment.
  - Supply $(\sigma(x-l,t))$ is the amount of traffic that can possibly enter the segment.

$$\delta(x,t) = \begin{cases} -\frac{v_f}{\rho_m}\rho(x,t)^2 + v_f\rho(x,t) & , \rho(x,t) < \frac{1}{2}\rho_m \\ \frac{1}{4}\rho_m v_f & , \frac{1}{2}\rho_m \le \rho(x,t) \le \rho_m \end{cases}$$

$$\sigma(x,t) = \begin{cases} \frac{1}{4}\rho_m v_f & , \rho(x,t) < \frac{1}{2}\rho_m \\ -\frac{v_f}{\rho_m}\rho(x,t)^2 + v_f\rho(x,t) & , \frac{1}{2}\rho_m \le \rho(x,t) \le \rho_m \end{cases}$$

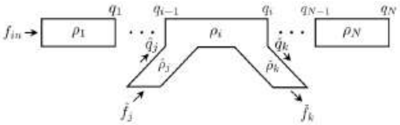
$$\boxed{q(x,t)) = \min(\delta(x,t)), \sigma(x-l,t))}$$

Greenshields: state-space representation

$$\dot{\rho}(x,t) = \frac{q(x,t) - q(x+l,t)}{l}$$
$$= \frac{\min(\delta(x,t)), \sigma(x-l,t)) - \min(\delta(x+l,t)), \sigma(x,t))}{l}$$

- how to remove the $\min()$?
- What if make $\delta(x,t)$ and $\theta(x,t)$ not piecewise?
- Yes!
  1. If all the segments are non-congested (i.e. $\rho(x,t) < \frac{1}{2}\rho_m$)
     $$\min(\delta(x,t)), \sigma(x-l,t)) = \delta(x,t) = -\frac{v_f}{\rho_m}\rho(x,t)^2 + v_f\rho(x,t)$$
  2. If all the segments are congested (i.e. $\frac{1}{2}\rho_m \leq \rho(x,t) \leq \rho_m$)
     $$\min(\delta(x,t)), \sigma(x-l,t)) = \sigma(x-l,t) = -\frac{v_f}{\rho_m}\rho(x-l,t)^2 + v_f\rho(x-l,t)$$

Greenshields: non-congested



1. The first cell: $i \in \mathcal{E} \backslash \mathcal{E}_I \cup \mathcal{E}_O, i = 1$

$$\dot{\rho}_i = \frac{f_{\text{in}} - q_i}{l} = \frac{f_{\text{in}}}{l} - \frac{v_f}{l}\rho_i + \delta \rho_i^2$$

2. The cell without on-ramps and off-ramps: $i \in \mathcal{E} \backslash \mathcal{E}_I \cup \mathcal{E}_O, i \neq 1$

$$\dot{\rho}_i = \frac{q_{i-1} - q_i}{l} = \frac{v_f}{l}\left(\rho_{i-1} - \rho_i\right) - \delta\left(\rho_{i-1}^2 - \rho_i^2\right)$$

3. With on-ramp
4. With off-ramp
5. With on-ramp and off-ramp
6. On-ramp
7. Off-ramp

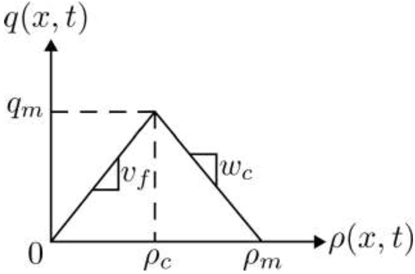
Greenshields: nonlinear state-space equation

$$\dot{x}(t) = \begin{bmatrix} A_1 & A_2 \\ O & A_3 \end{bmatrix} x(t) + f(x) + B_{\mathrm{u}} u(t)$$

Parameter	Description
$\boldsymbol{A}_1 \in \mathbb{R}^{N \times N}$, $\boldsymbol{A}_2 \in \mathbb{R}^{N \times (N_I + N_O)}$, $\boldsymbol{A}_3 \in \mathbb{R}^{(N_I+N_O) \times (N_I+N_O)}$	$\boldsymbol{A}_1 = \begin{bmatrix} -\frac{v_f}{l} & 0 & 0 & \cdots & 0 \\ \frac{v_f}{l} & -\frac{v_f}{l} & 0 & \cdots & 0 \\ 0 & \frac{v_f}{l} & -\frac{v_f}{l} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -\frac{v_f}{l} \end{bmatrix}$ $\boldsymbol{A}_2(i,j) = \begin{cases} \frac{v_f}{l}, & \text{if } i \in \boldsymbol{\mathcal{E}}_I,\ j \in \hat{\boldsymbol{\mathcal{E}}} \\ -\frac{\alpha(\bar{j}) v_f}{l}, & \text{if } i \in \boldsymbol{\mathcal{E}}_O,\ j = N_I + \bar{j},\ \bar{j} \in \check{\boldsymbol{\mathcal{E}}} \\ 0, & \text{otherwise} \end{cases}$ $\boldsymbol{A}_3(i,j) = \begin{cases} -\frac{v_f}{l}, & \text{if } i = j, i \in \hat{\boldsymbol{\mathcal{E}}} \\ \frac{\alpha(\bar{i}) v_f}{l}, & \text{if } i = j,\ i = N_I + \bar{i}, \bar{i} \in \check{\boldsymbol{\mathcal{E}}} \\ 0, & \text{otherwise.} \end{cases}$
$\boldsymbol{f} : \mathbb{R}^n \to \mathbb{R}^n$	$f_i(\boldsymbol{x}) = \begin{cases} \delta x_i^2, & \text{if } i \in \boldsymbol{\mathcal{E}} \setminus \boldsymbol{\mathcal{E}}_I \cup \boldsymbol{\mathcal{E}}_O,\ i = 1 \\ \delta \left(x_i^2 - x_{i-1}^2 \right), & \text{if } i \in \boldsymbol{\mathcal{E}} \setminus \boldsymbol{\mathcal{E}}_I \cup \boldsymbol{\mathcal{E}}_O,\ i \neq 1 \\ \delta \left(x_i^2 - x_{i-1}^2 - x_j^2 \right), & \text{if } i \in \boldsymbol{\mathcal{E}}_I \setminus \boldsymbol{\mathcal{E}}_I \cap \boldsymbol{\mathcal{E}}_O,\ j = N + \bar{j},\ \bar{j} \in \hat{\boldsymbol{\mathcal{E}}} \\ \delta \left(x_i^2 - x_{i-1}^2 + \alpha(\bar{j}) x_j^2 \right), & \text{if } i \in \boldsymbol{\mathcal{E}}_O \setminus \boldsymbol{\mathcal{E}}_I \cap \boldsymbol{\mathcal{E}}_O,\ j = N + N_I + \bar{j},\ \bar{j} \in \check{\boldsymbol{\mathcal{E}}} \\ \delta \left(x_i^2 - x_{i-1}^2 - x_j^2 + \alpha(\bar{k}) x_k^2 \right), & \text{if } i \in \boldsymbol{\mathcal{E}}_I \cap \boldsymbol{\mathcal{E}}_O,\ j = N + \bar{j},\ \bar{j} \in \hat{\boldsymbol{\mathcal{E}}},\ k = N + N_I + \bar{k},\ \bar{k} \in \check{\boldsymbol{\mathcal{E}}} \\ \delta x_i^2, & \text{if } i = N + \bar{i},\ \bar{i} \in \hat{\boldsymbol{\mathcal{E}}} \\ -\alpha(\bar{i}) \delta x_i^2, & \text{if } i = N + N_I + \bar{i},\ \bar{i} \in \check{\boldsymbol{\mathcal{E}}} \end{cases}$
$\boldsymbol{B}_{\mathbf{u}} \in \mathbb{R}^{n \times (1+N_I+N_O)}$	$\boldsymbol{B}_{\mathbf{u}}(i,j) = \begin{cases} \frac{1}{l}, & \text{if } i = j = 1,\ i \in \boldsymbol{\mathcal{E}} \\ \frac{1}{l}, & \text{if } i = N + \bar{i},\ j = 1 + \bar{i},\ \bar{i} \in \hat{\boldsymbol{\mathcal{E}}} \\ -\frac{1}{l}, & \text{if } i = N + N_I + \bar{i},\ j = 1 + N_I + \bar{i},\ \bar{i} \in \check{\boldsymbol{\mathcal{E}}} \\ 0, & \text{otherwise} \end{cases}$
$\boldsymbol{u} \in \mathbb{R}^{1+N_I+N_O}$	$\boldsymbol{u}(t) = \begin{bmatrix} f_{\mathrm{in}} & \hat{f}_1 & \hat{f}_2 & \cdots & \hat{f}_{N_I} & \check{f}_1 & \check{f}_2 & \cdots & \check{f}_{N_O} \end{bmatrix}^\top$

Standard CTM: Triangular fundamental diagram

- Homework
- Triangular fundamental diagram



$$Q(\rho) = \begin{cases} v_f \rho & , \rho < \rho_c \\ w_c(\rho - \rho_m) & , \rho_c \leq \rho \leq \rho_m \end{cases}$$

- Hint: follow the same structure as Greenshields, what is the demand equation and supply equation?

Sensor placement

For the given state-space and measurement model:

$$\begin{aligned} \boldsymbol{x}[k+1] &= \boldsymbol{A}\boldsymbol{x}[k] + \boldsymbol{G}\boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},k) + \boldsymbol{B}_{\mathbf{u}}\boldsymbol{u}[k] \\ \boldsymbol{y}[k] &= \boldsymbol{\Gamma}\boldsymbol{C}\boldsymbol{x}[k]. \end{aligned}$$

the observability Gramian can be written as:

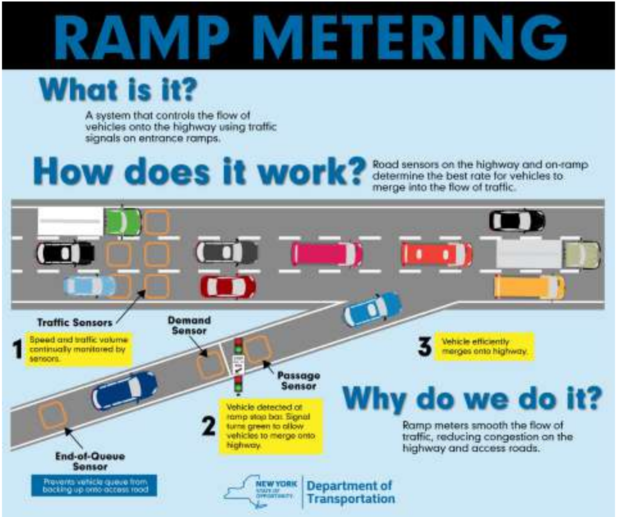
$$\boldsymbol{W}_o\left(\boldsymbol{\gamma},\boldsymbol{x}_0\right) := \boldsymbol{J}_w^{\top}\left(\boldsymbol{\gamma},\boldsymbol{x}_0\right)\boldsymbol{J}_w\left(\boldsymbol{\gamma},\boldsymbol{x}_0\right),$$

where $\boldsymbol{J}_w^{\top}\left(\boldsymbol{\gamma},\boldsymbol{x}_0\right)$ is a Jacobian matrix. We further minimize the determinant or trace of the Gramian Matrix to maximize the degree of observability by solving the below convex IP:

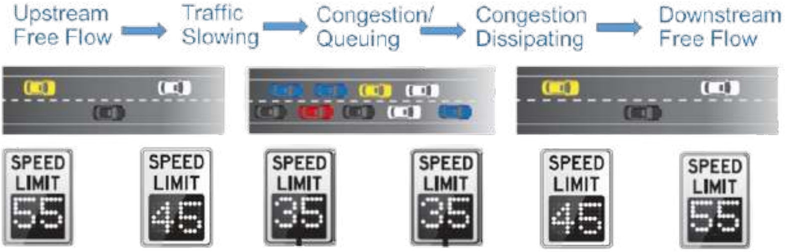
$$(\mathbf{P1})\kappa = \underset{\gamma}{\text{minimize}} \begin{cases} -\det\left(\boldsymbol{W}_o\left(\boldsymbol{\gamma},\hat{\boldsymbol{x}}_0\right)\right), \\ -\operatorname{trace}\left(\boldsymbol{W}_o\left(\boldsymbol{\gamma},\hat{\boldsymbol{x}}_0\right)\right), \end{cases}$$

subject to $\quad \gamma \in \mathcal{G}_{\gamma}, \boldsymbol{\gamma} \in \{0,1\}^p.$

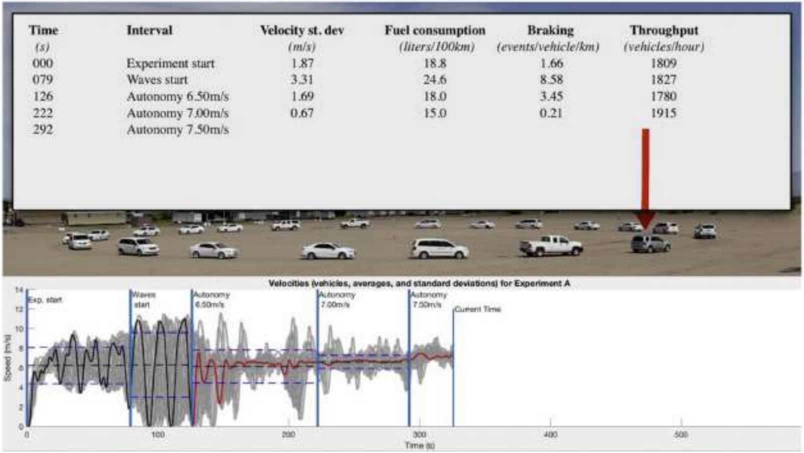
Ramp metering



VSL (variable speed limit) control



CAV traffic control



Time (s)	Interval	Velocity st. dev (m/s)	Fuel consumption (liters/100km)	Braking (events/vehicle/km)	Throughput (vehicles/hour)
000	Experiment start	1.87	18.8	1.66	1809
079	Waves start	3.31	24.6	8.58	1827
126	Autonomy 6.50m/s	1.69	18.0	3.45	1780
222	Autonomy 7.00m/s	0.67	15.0	0.21	1915
292	Autonomy 7.50m/s				

Figure: Self-driving cars experiment demonstrates dramatic improvements in traffic flow. Source:https://www.youtube.com/watch?v=2mBjYZTeaTc&t=4s