Name:

Consider a function $f(x) : \mathbb{R}^n \rightarrow \mathbb{R}$. The linear approximation (1st-order Taylor approximation) of $f(x)$ around point x_0 is given as

$$f_{\text{lin}}(x) = f(x_0) + \nabla f(x_0)^T (x - x_0)$$

whereas the quadratic approximation (2nd-order Taylor approximation) of $f(x)$ around point x_0 is defined as

$$f_{\text{quad}}(x) = f(x_0) + \nabla f(x_0)^T (x - x_0) + \frac{1}{2}(x - x_0)^T \nabla^2 f(x_0)(x - x_0).$$

Compute the 1st and 2nd order approximations of this function

- $f(x) = x_1^3 + x_2^2 + x_3^2 + 2e^{x_1} x_3 x_2 + 3x_1 x_2 x_3 + x_1^2 x_2 + 10$ around $x_0 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$