VANDERBILT UNIVERSITY
CE 5999
INTRO TO OPTIMIZATION

HOMEWORK # 2
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February 3, 2023

Due date of the homework: February 12, midnight. Justify your answers carefully.

1. Reading material: Appendices A.4 and A.5.
2. Are the following sets convex or nonconvex? Justify your answer.
   (a) A sphere with one point on the boundary removed.
   (b) A cube with one vertex removed.
   (c) A cube with one, non-vertex point on the boundary removed.
   (d) A sphere with one interior point removed.
3. Consider two points y and z in $\mathbb{R}^n$. Prove that the set of points that are closer to y than to z is a polyhedron. Write the set in the form $\{x \mid Ax \preceq b\}$.
   Hint: The set of points closer to y than z is written as follows:
   $$S = \{x \in \mathbb{R}^n \mid \|x - y\|_2 \leq \|x - z\|_2\}.$$
   Take the square of the previous inequality and express the norm using the inner product.
4. Prove that the set of $n \times n$ symmetric matrices is a subspace.
   Hint: Use the definition of subspace.
5. Boyd-Vandenberghe 2.7.
6. Boyd-Vandenberghe 2.12.
7. Boyd-Vandenberghe 2.28.
   Hint: Use the following result we covered in class: A symmetric $n \times n$ matrix is positive semidefinite if and only if all its principal minors are nonnegative. Check out the slides on what principal minors are.