

The objective of this homework is to test your understanding of the content of Module 3. The date of this homework is: **Sunday, September 13th, 2020**. You have to upload a scanned version of your solutions on Blackboard. If you don't have a scanner around you, you can use Cam Scanner—a mobile app that scans images in a neat way, as if they're scanned through a copier.

For questions related to the homework, please email the two teaching assistants: (a) Wasim (email: wasim.dipon@my.utsa.edu) and (b) Yuvaraj (email: yuvaraj.munian@utsa.edu).

You can send your question to both of them on the same email. Feel free to CC me as well, but that is not necessary.

1. For the given matrices,

$$A = \begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 3 & x \\ y & 1 \end{bmatrix}$$

- (i) Compute the determinants of A, B, C. (5 points)
 - (ii) Find all values of x and y that make $AB = AC$, but B is not equal to C. (15 points)
2. Let A be a 2x2 matrix (You can use A matrix from Question 1). Determine matrices performing the following operations that alter matrix A. (5 points)
- (i) Interchange rows 1 and 2;
 - (ii) Multiply (scale) column 2 by 4, replace column 2;
 - (iii) Add row 1 to row 2 and replace row 2;
 - (iv) Multiply (scale) row 1 by 2 and row 2 by -1.

3. Find the inverse of the following matrices X, Y, Z. (15 points)

$$X = \begin{bmatrix} 4 & -2 \\ 2 & 0 \end{bmatrix} \quad Y = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad Z = \begin{bmatrix} 1 & -6 \\ -5 & 6 \end{bmatrix}$$

[Hint: If A is the 2x2 matrix given by, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and if $ad - bc \neq 0$, the inverse is given by

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}]$$

4. Divide the complex numbers and express the result in standard form, $a + bi$, where a and b are fractions in lowest terms. (10 points)

$$\frac{5 + 7i}{3 + 2i}$$

5. Write the complex number in polar form with argument θ between 0 and 2π (10 points)

$$-5 + 5\sqrt{3}i$$

[Hint : A complex number $z = a + bi$ has the polar form $z = r(\cos\theta + i\sin\theta)$, where $r = |z| = \sqrt{a^2 + b^2}$, $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ for $a > 0$ and $\theta = \tan^{-1}\left(\frac{b}{a}\right) + \pi$ or $\theta = \tan^{-1}\left(\frac{b}{a}\right) + 180^\circ$ for $a < 0$. The r and θ are the modulus and argument of z .]

6. Sketch the set in the complex plane. (10 points)

$$\{2 \leq |z| \leq 5\}$$

[Hint: Modulus the complex of the $r = |z| = \sqrt{a^2 + b^2}$]

7. Calculate the following limits if they exist. If they do not exist, then indicate whether they are $+\infty$, $-\infty$ or undefined. (10 points)

(i) $\lim_{x \rightarrow 0} \frac{4}{x - 1}$

(ii) $\lim_{x \rightarrow 3} \frac{4x^2}{2 - x}$

8. Calculate the derivate of each of these functions directly from the definition. (20 points)

(i) $f(x) = \frac{1}{(2x+1)}$

(ii) $f(x) = 2x^2 + 5x + 4$