

EE 1322: Intro to Electrical & Computer Engineering

Module 5: Intro to Electric Circuits

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1. Introduction and basics
2. Units and conversion
3. Basic circuit components
4. Voltage and current laws

Part 1:

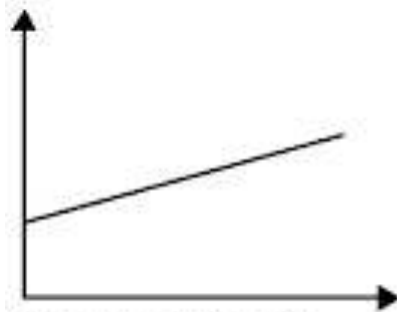
Introduction and Basics

Problem solving; linear vs nonlinear phenomena

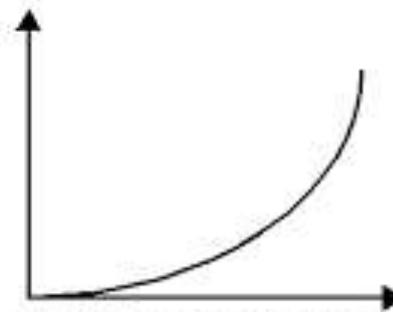
- All engineers share a considerable amount of common ground, particularly when it comes to problem solving
- Circuit analysis: at the core of **problem solving from an engineering perspective**

Linear vs. Nonlinear?

- Linear problems are inherently more easily solved than nonlinear ones
- We often seek reasonably accurate linear approximations
- Linear models are more easily manipulated and understood

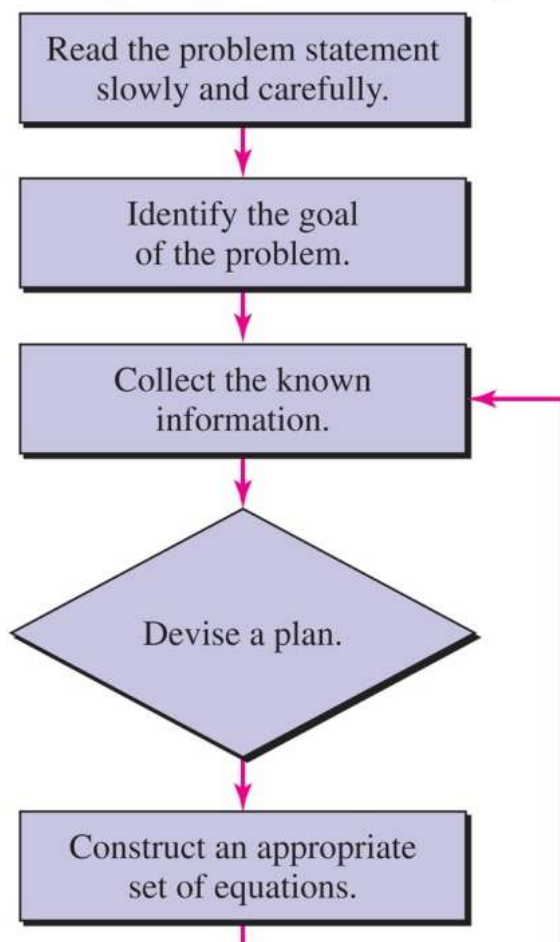
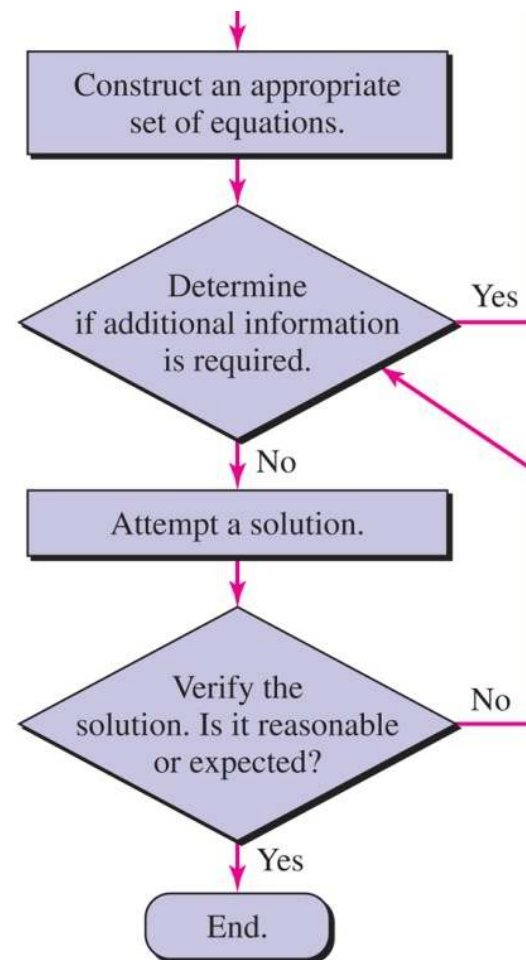


Linear relationship
Number of personnel vs. employee costs



Nonlinear relationship
Population growth over time

Problem solving 101



Part 2:

Units and Conversion

The SI (International System of Units) system

Base units:

- meter (m), kilogram (kg), second (s), ampere (A)
- also: kelvin, mole, and candela

Derived units:

- work or energy: joule (J)
- power (rate of doing work): watt (W)
- $1 \text{ W} = 1 \text{ J/s}$

SI Base Units		
Quantity	SI base unit	Symbol
Length	meter	m
Mass	kilogram	kg
Temperature	kelvin	K
Time	second	s
Amount of substance	mole	mol
Luminous intensity	candela	cd
Electric current	ampere	A

SI: Units + prefixes

Any measurement can be expressed in terms of a unit, or a unit with a “prefix” modifier.

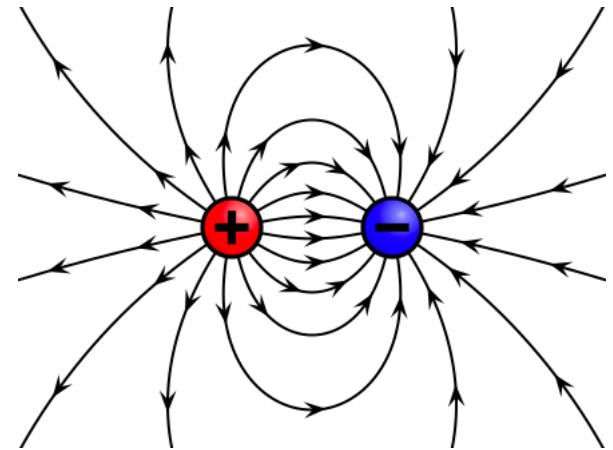
Commonly Used Metric Prefixes

Prefix	Symbol	Meaning	Factor
mega	M	1 million times larger than the unit it precedes	10^6
kilo	k	1000 times larger than the unit it precedes	10^3
deci	d	10 times smaller than the unit it precedes	10^{-1}
centi	c	100 times smaller than the unit it precedes	10^{-2}
milli	m	1000 times smaller than the unit it precedes	10^{-3}
micro	μ	1 million times smaller than the unit it precedes	10^{-6}
nano	n	1 billion times smaller than the unit it precedes	10^{-9}
pico	p	1 trillion times smaller than the unit it precedes	10^{-12}

Example: $12.3 \text{ mW} = 0.0123 \text{ W} = 1.23 \times 10^{-2} \text{ W}$

Charge

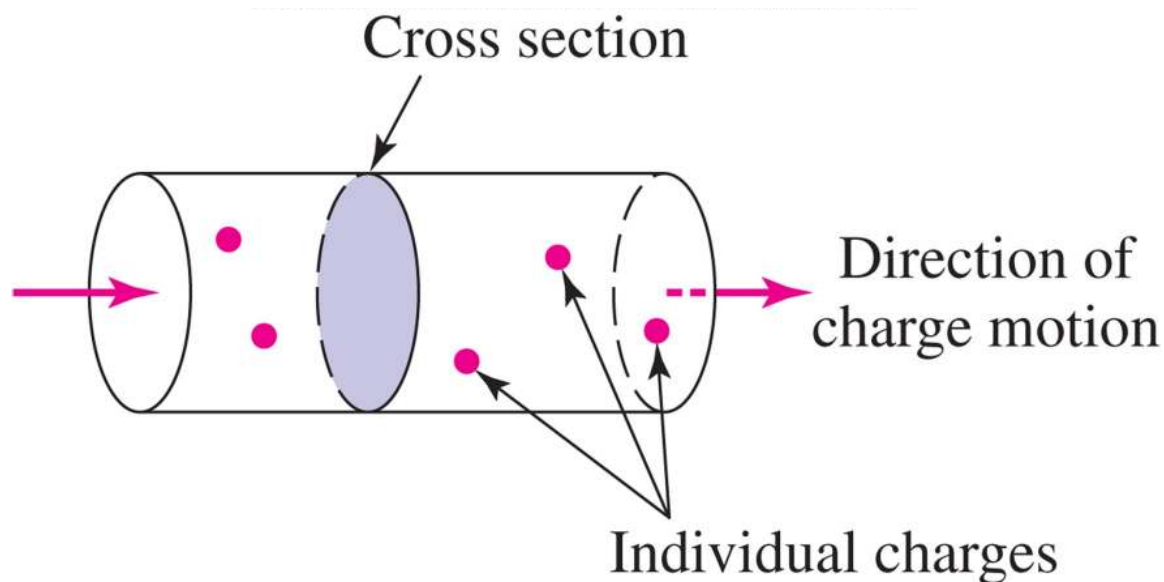
- Electric charge: physical property of matter
 - that causes it to experience a force when placed in an electromagnetic field
 - can be positive or negative
- Charge is *conserved*: it is neither created nor destroyed
- Symbol: Q or q ; units are coulomb (C)
- The smallest charge, the *electronic charge*, is carried by an electron (-1.602×10^{-19} C) or a proton ($+1.602 \times 10^{-19}$ C)
- In most circuits, the charges in motion are electrons



Current and charge

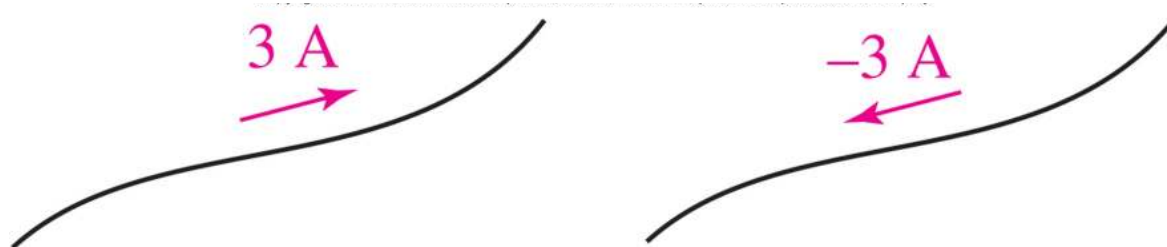
Current is the rate of charge flow:

1 ampere = 1 coulomb/second (or $1 \text{ A} = 1 \text{ C/s}$)

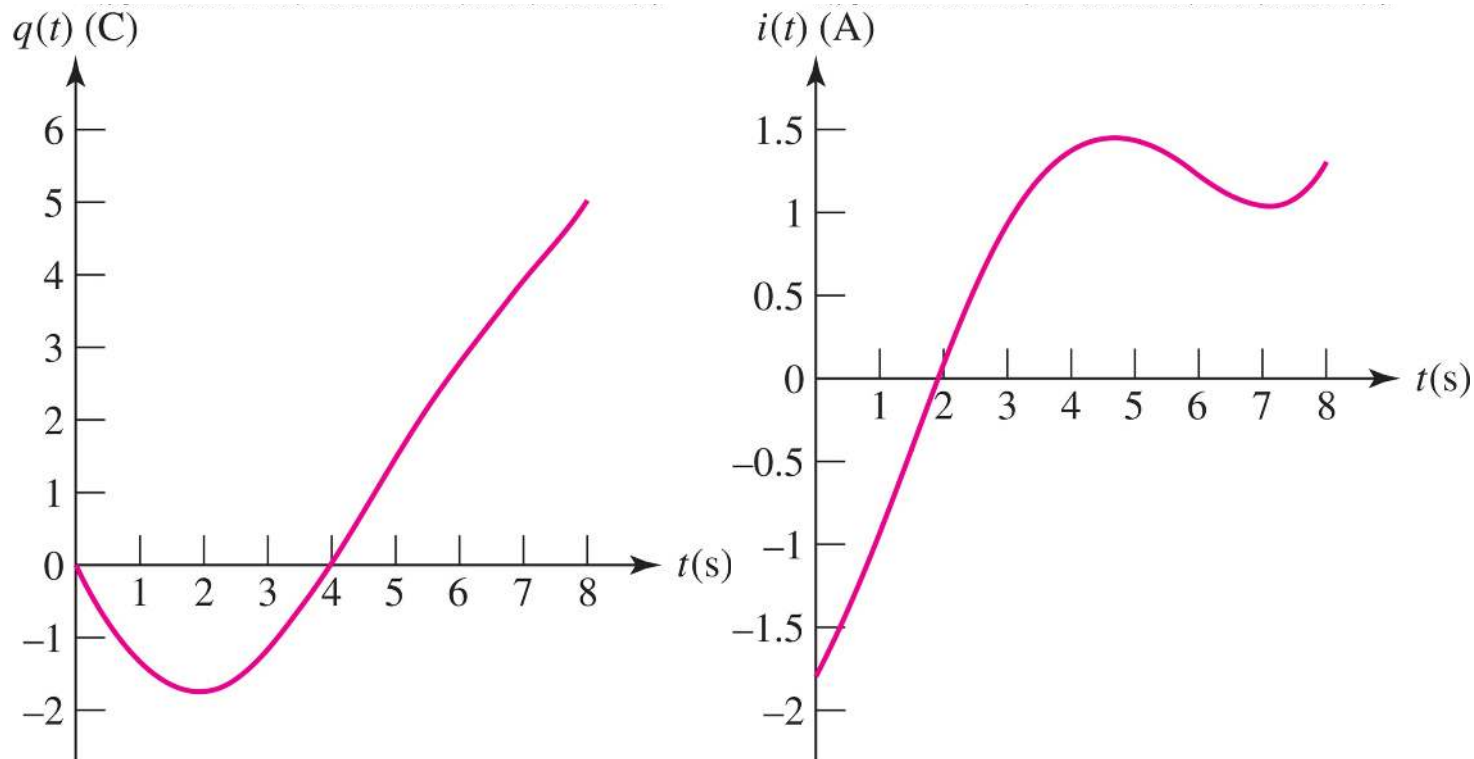


Current and charge

- Current (designated by I or i) is the rate of flow of charge
- Current must be designated with both a direction and a magnitude
- These two currents are the same:



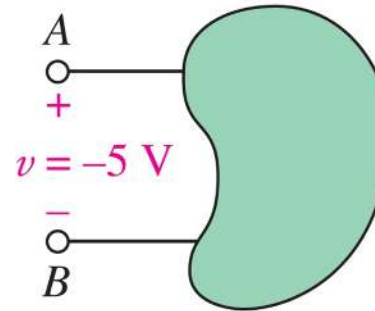
Current and charge



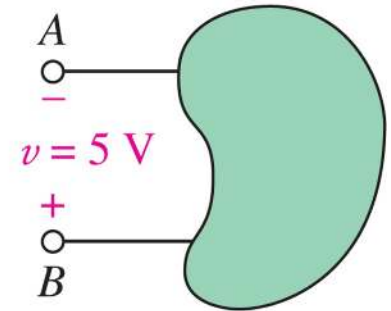
$$I(t) = \frac{dq(t)}{dt} = \dot{q}(t)$$

Voltage

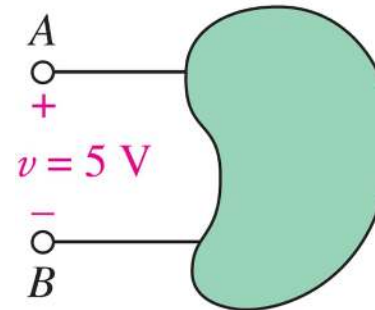
- When 1 J of work is required to move 1 C of charge from A to B, there is a voltage of 1 volt between A and B
- Voltage (V or v) across an element requires both a magnitude and a polarity
- Example: (a)=(b), (c)=(d)



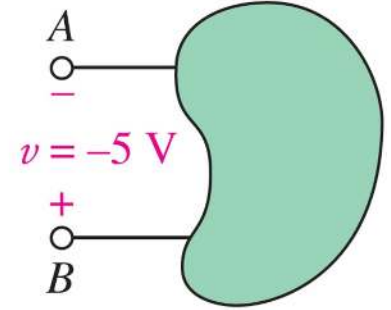
(a)



(b)



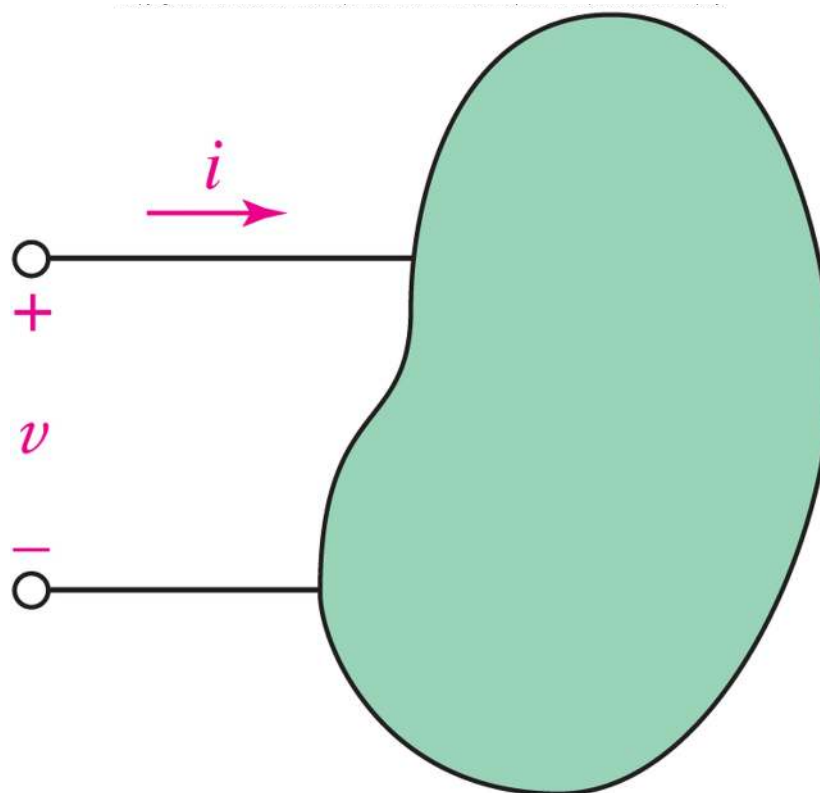
(c)



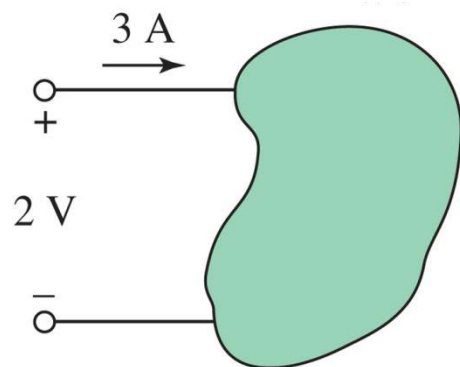
(d)

Power $P(t) = V(t)I(t)$

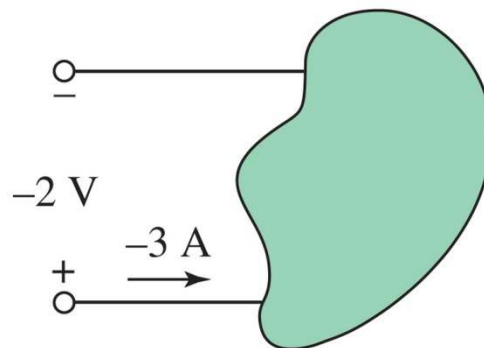
- The power required to push a current i (C/s) into a voltage v (J/C) is $p = vi$ (J/s = W)
- When power is positive, the element is *absorbing* energy
- When power is negative, the element is *supplying* energy



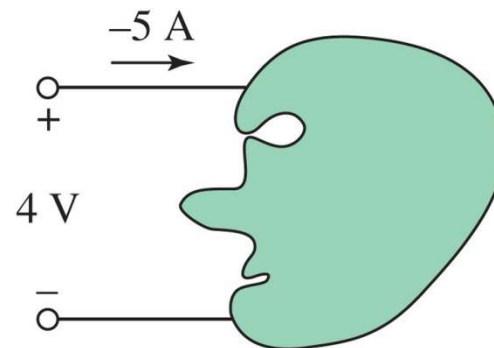
Example 1: power



(a)



(b)



(c)

How much power is absorbed by the three elements above?

$$P_a = +6 \text{ W}, P_b = +6 \text{ W}, P_c = -20 \text{ W}.$$

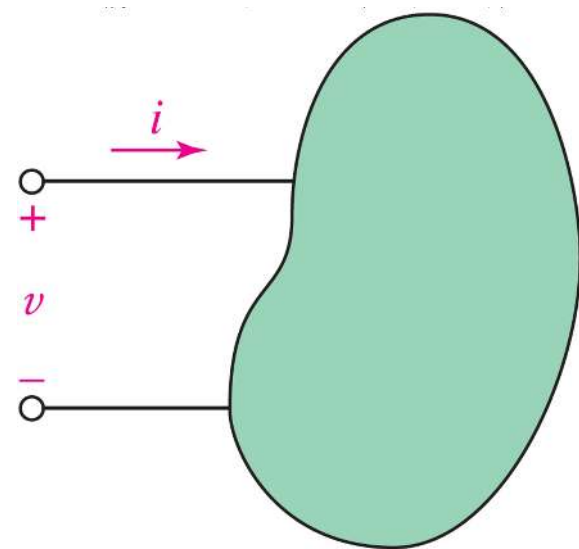
(Note: (c) is actually supplying power)

Part 3:

Basic Circuit Components

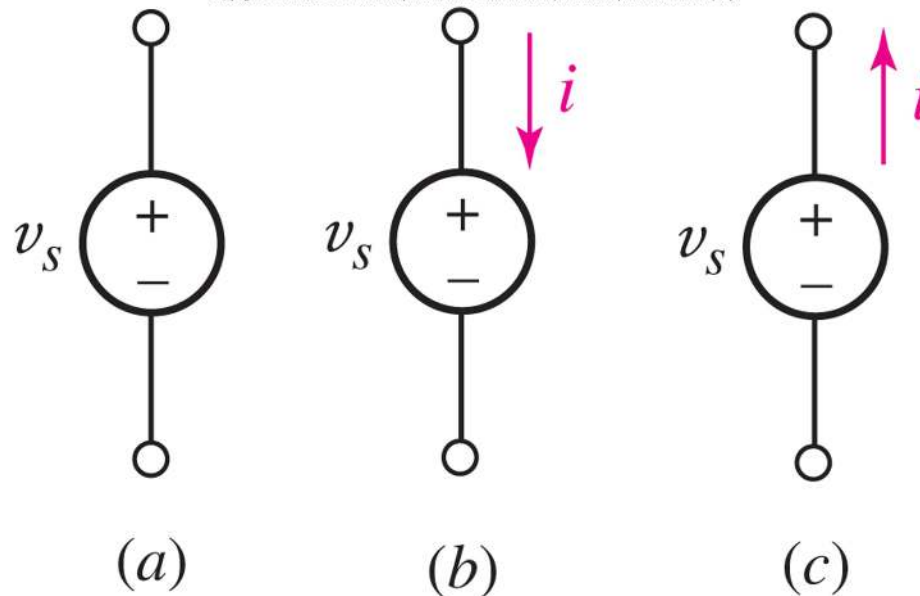
Circuit elements

- A circuit element usually has two terminals (sometimes three or more)
- The relationship between the voltage v across the terminals and the current i through the device defines the circuit element model



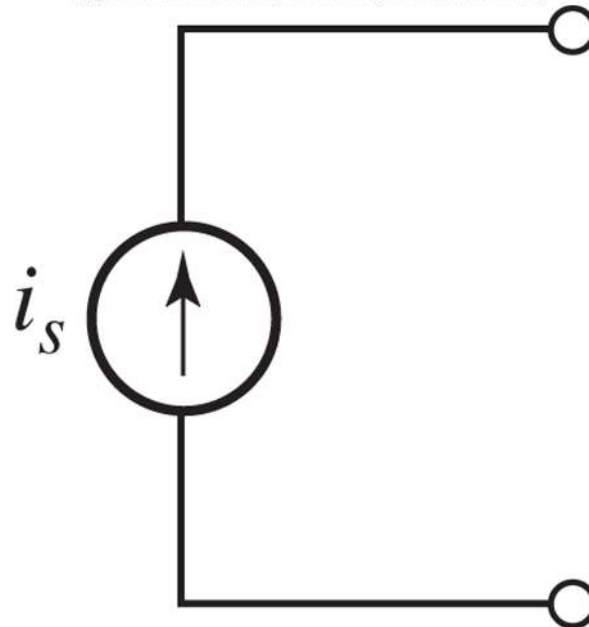
Voltage sources

- An ideal voltage source is a circuit element that will maintain the specified voltage v_s across its terminals
- The current will be determined by other circuit elements



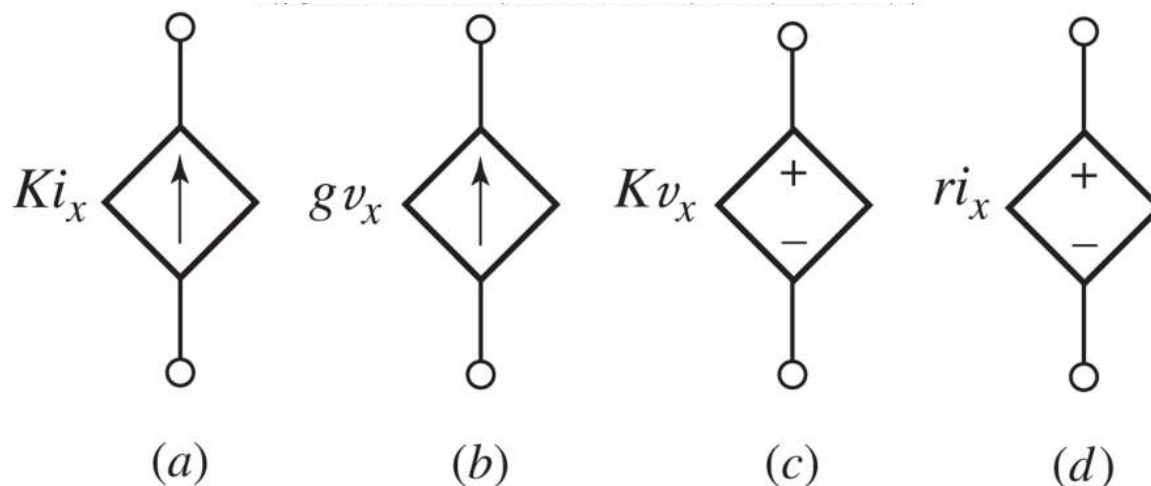
Current sources

- An ideal current source is a circuit element that maintains the specified current flow i_s through its terminals
- The voltage is determined by other circuit elements



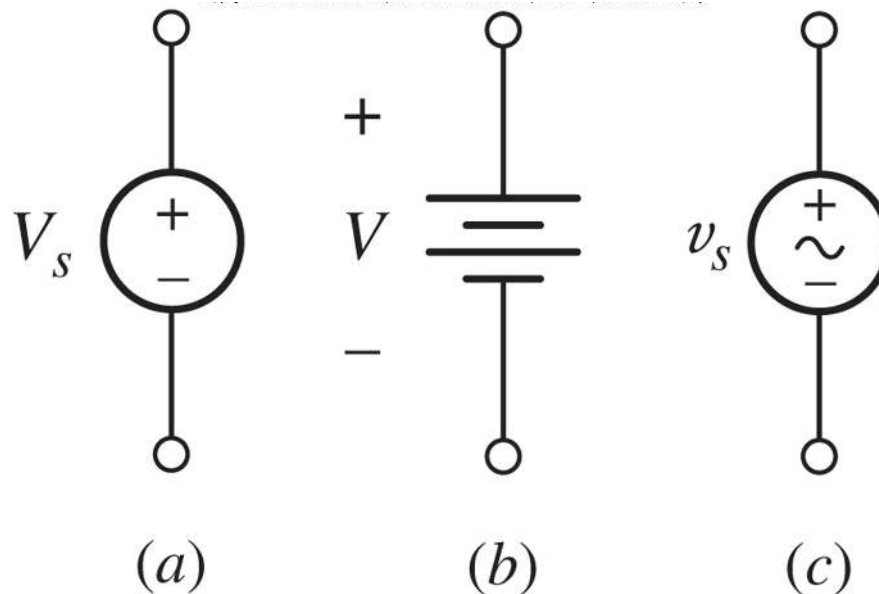
Dependent sources

- Dependent current sources (a) and (b) maintain a *current* specified by another circuit variable.
- Dependent voltage sources (c) and (d) maintain a *voltage* specified by another circuit variable.



Battery as voltage source

- A voltage source is an idealization (no limit on current) and generalization (voltage can be time-varying) of a battery
- A battery supplies a constant “dc” voltage V but in practice a battery has a maximum power



Ohm's Law: resistance

- A (linear) resistor is an element for which

$$V(t) = RI(t)$$

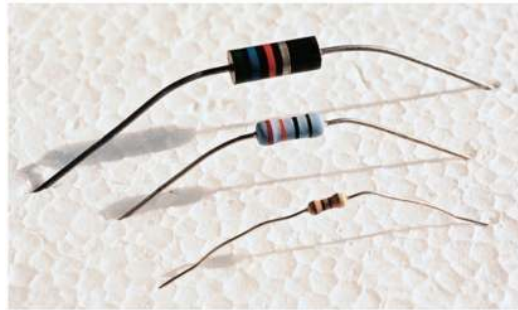
where the constant R is a resistance

- The equation is known as “Ohm’s Law”
- The unit of resistance is ohm (Ω)



Resistors

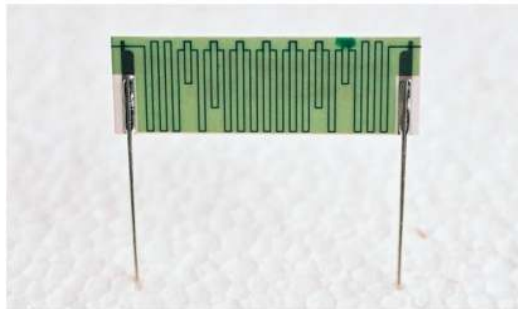
(a) typical resistors (b) power resistor
(c) a 10 T Ω resistor (d) circuit symbol



(a)



(b)



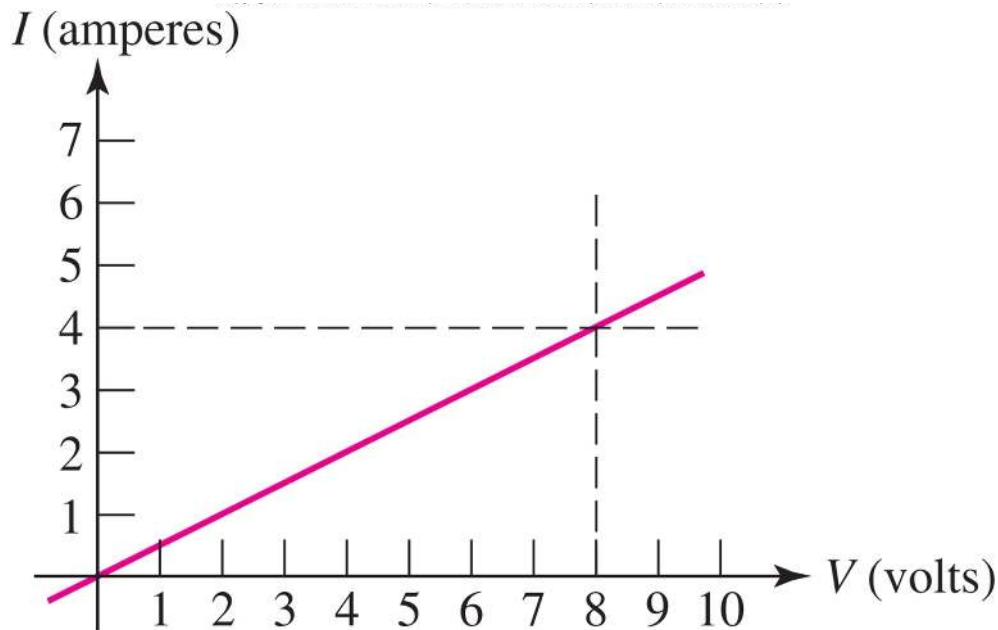
(c)



(d)

The I-V graph for a resistor

For a resistor, the plot of current versus voltage is a straight line:



In this example, the slope is 4 A / 8 V or $0.5 \Omega^{-1}$.

This is the graph for a 2 ohm resistor.

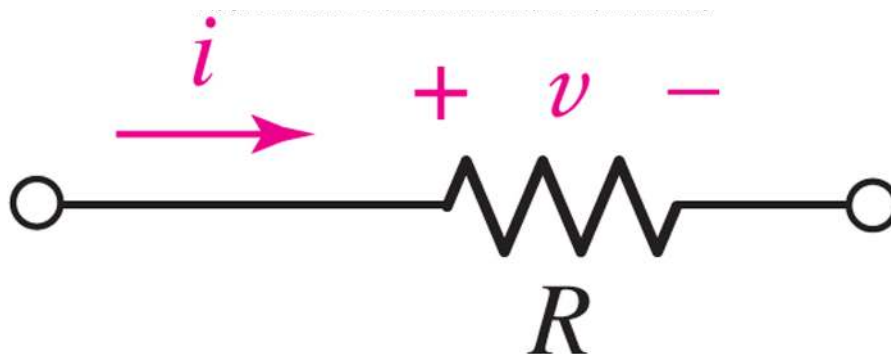
Power absorption

Resistors absorb power according to this law:

$$P = VI = \frac{V^2}{R} = I^2 R$$

Positive power means the device is absorbing energy

Power is always positive for a resistor!



Example: resistor power

A $560\ \Omega$ resistor is connected to a circuit which causes a current of $42.4\ \text{mA}$ to flow through it. Calculate the voltage across the resistor and the power it is dissipating.

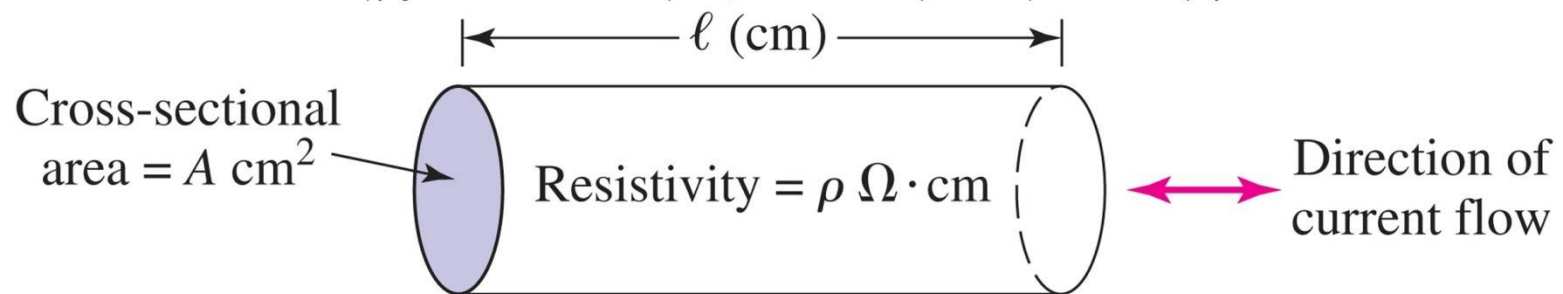
$$V = IR = (0.0424)(560) = 23.7V$$

$$P = I^2 R = (0.0424)^2(560) = 1.007W$$

Wire gauge and resistivity

The resistance of a wire is determined by the resistivity of the conductor as well as the geometry:

$$R = \frac{\rho l}{A}$$



In most cases, the resistance of wires can be assumed to be 0 ohms.

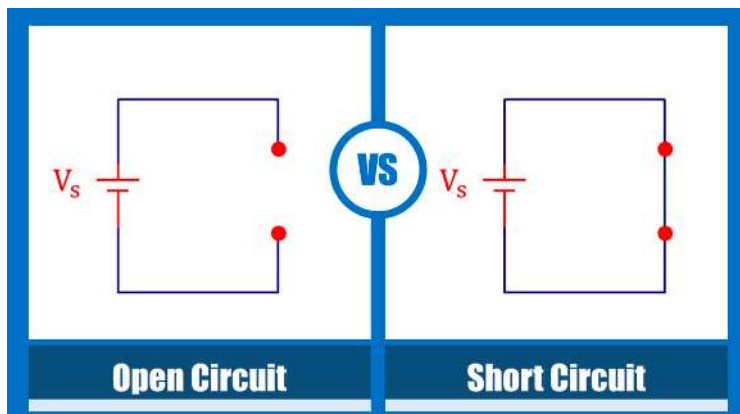
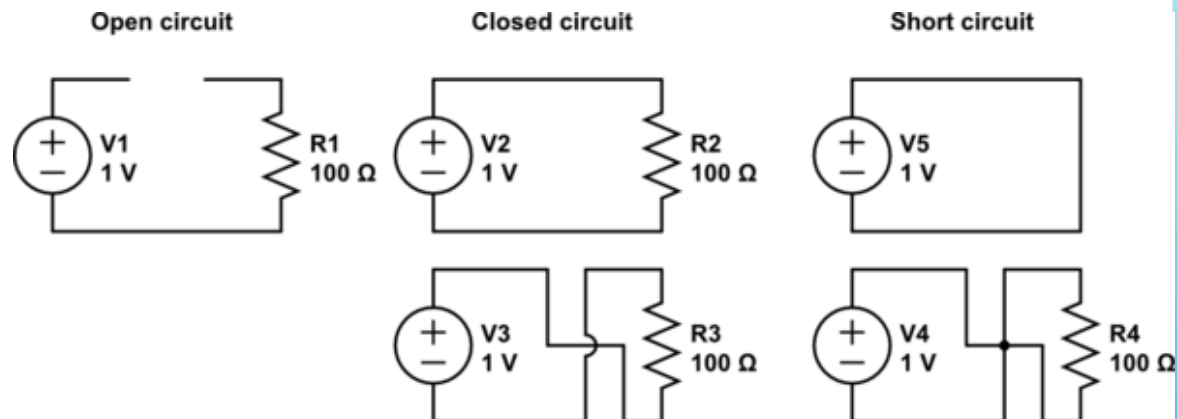
Conductance

- We sometimes prefer to work with *the reciprocal of resistance* ($1/R$), which is called conductance (symbol G , unit siemens (S))
- A resistor R has conductance $G=1/R$
- The i - v equation (i.e. Ohm's law) can be written as

$$I = GV$$

Open and short circuits

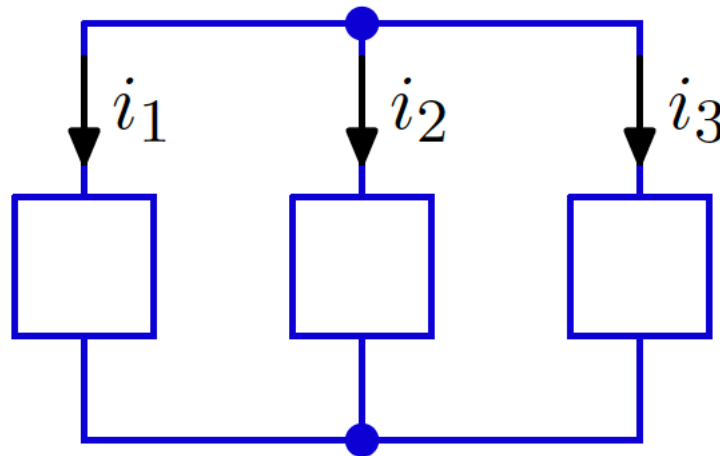
- An open circuit between A and B means $i=0$.
- *Voltage across* an open circuit: any value.
- An open circuit is equivalent to $R = \infty \Omega$.
- A short circuit between A and B means $v=0$.
- *Current through* a short circuit: any value.
- A short circuit is equivalent to $R = 0 \Omega$.



Part 4:

Voltage and Current Laws

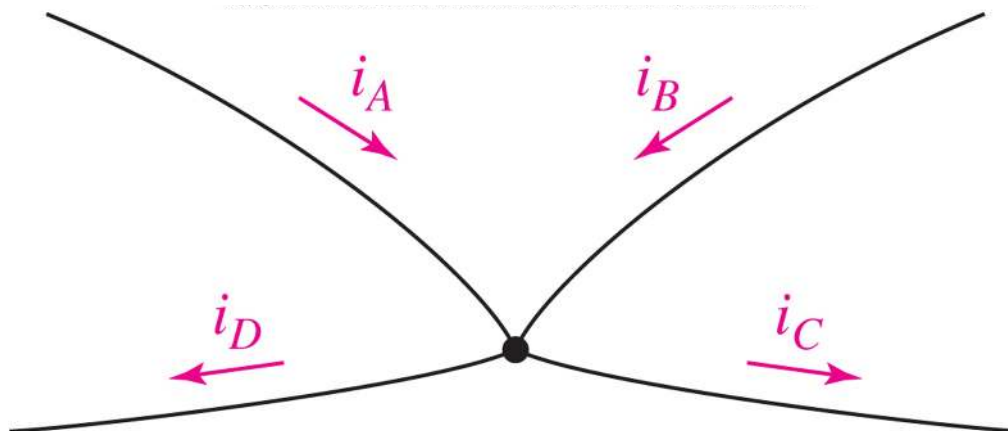
Nodes, paths, loops, branches



- Circuits are made up of elements connected by nodes.
- Above circuit has: three elements, two nodes, three branches.
- A *path* is a sequence of nodes
- a *loop* is a closed (circular) path

Kirchhoff's Current Law (KCL)

KCL: The algebraic sum of the currents entering any node is zero.



$$i_A + i_B + (-i_C) + (-i_D) = 0$$

KCL: Alternative Forms

- Current *IN* is zero:

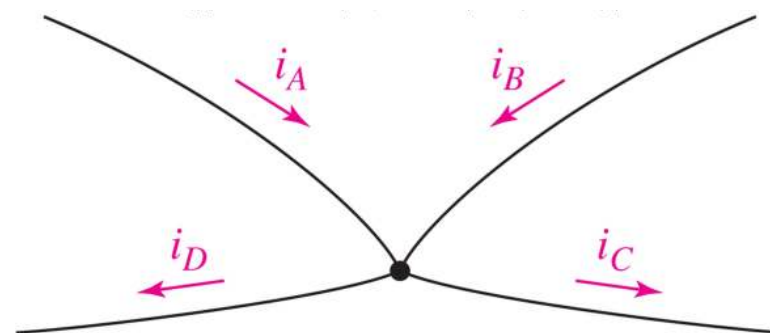
$$i_A + i_B + (-i_C) + (-i_D) = 0$$

- Current *OUT* is zero:

$$(-i_A) + (-i_B) + i_C + i_D = 0$$

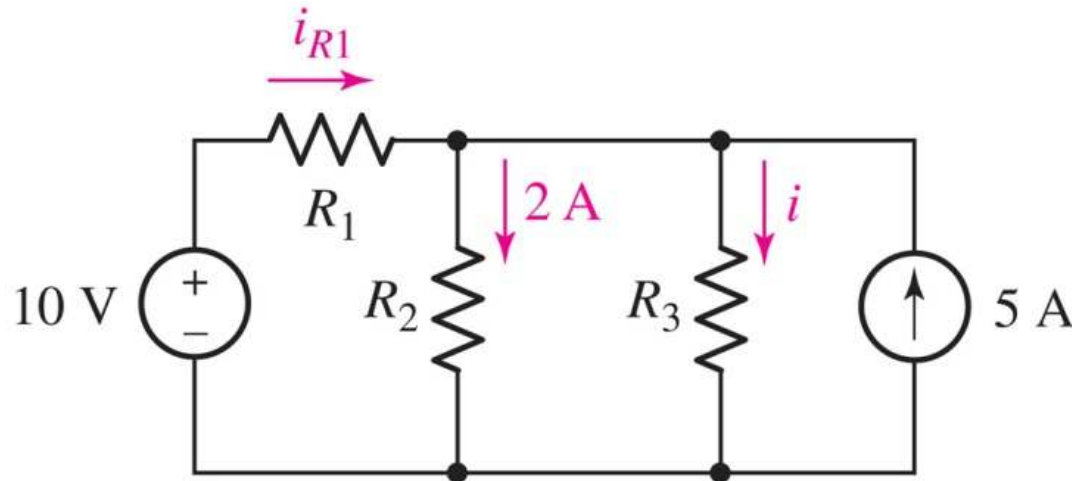
- Current *IN=OUT*:

$$i_A + i_B = i_C + i_D$$



Example: KCL application

Find the current through resistor R_3 if it is known that the voltage source supplies a current of 3 A.

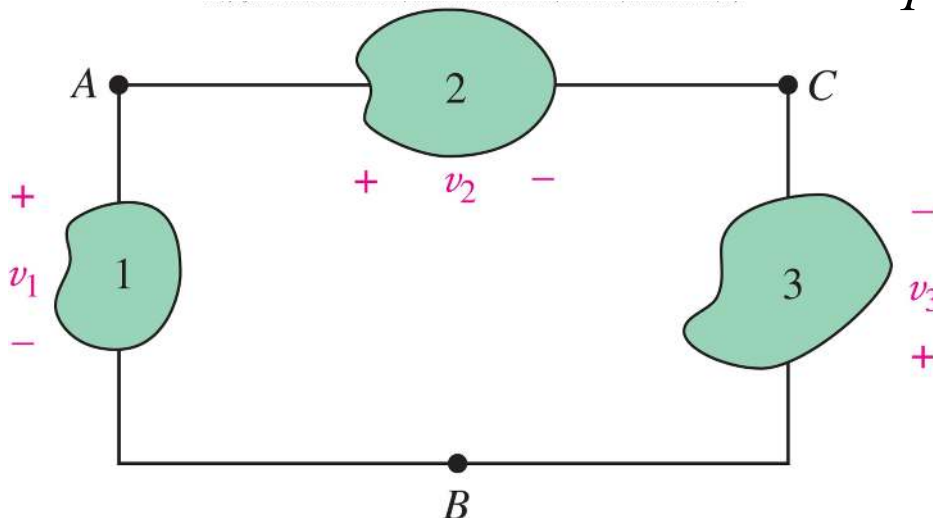


Answer: $i = 6\text{ A}$

Kirchhoff's Voltage Law (KVL)

KVL: The algebraic sum of the voltages around any closed path is zero.

$$v_1 + (-v_2) + (-v_3) = 0$$



KVL: Alternative Forms

- Sum of *RISES* is zero (clockwise from B):

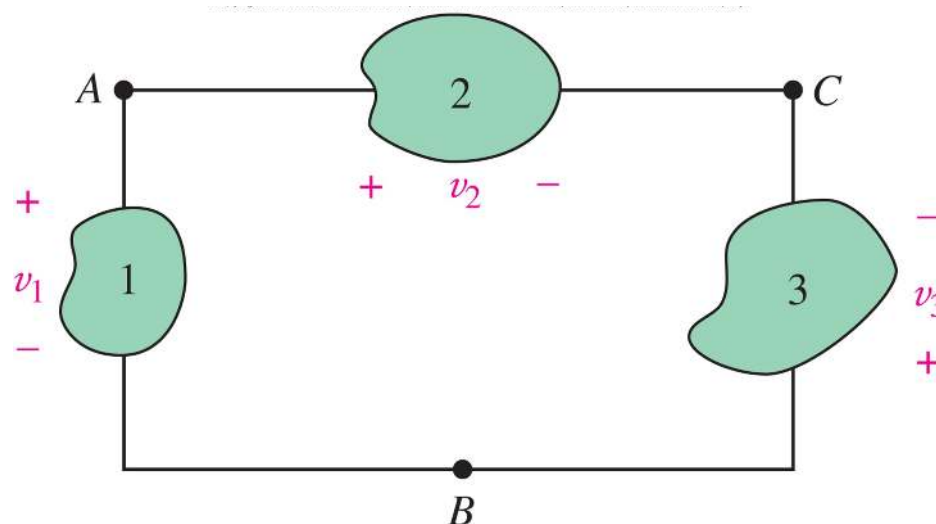
$$v_1 + (-v_2) + v_3 = 0$$

- Sum of *DROPS* is zero (clockwise from B):

$$(-v_1) + v_2 + (-v_3) = 0$$

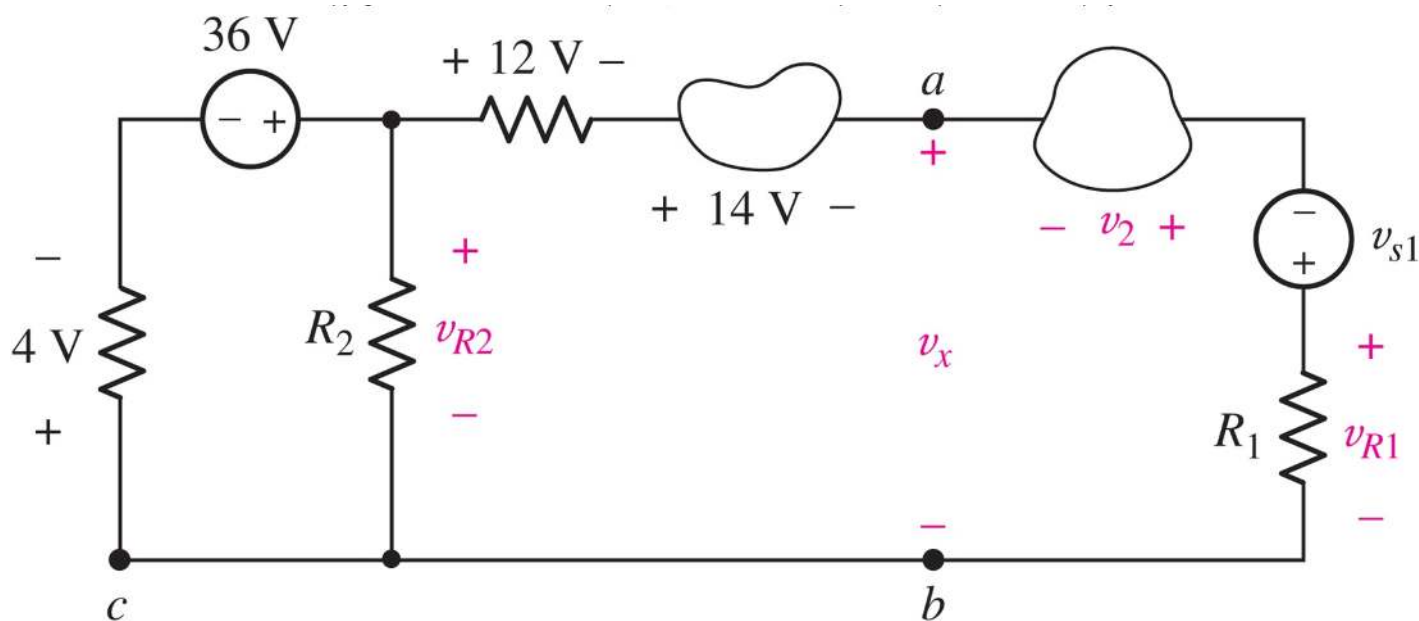
- Two paths, same voltage (A to B):

$$v_1 = (-v_3) + v_2$$



Example: Applying KVL

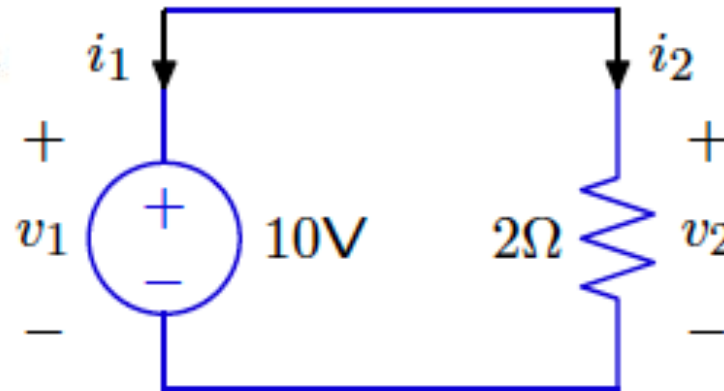
Find v_{R2} (the voltage across R_2) and the voltage v_x .



Answer: $v_{R2} = 32\text{ V}$ and $v_x = 6\text{ V}$.

Example: Applying KVL, KCL, Ohm's Laws

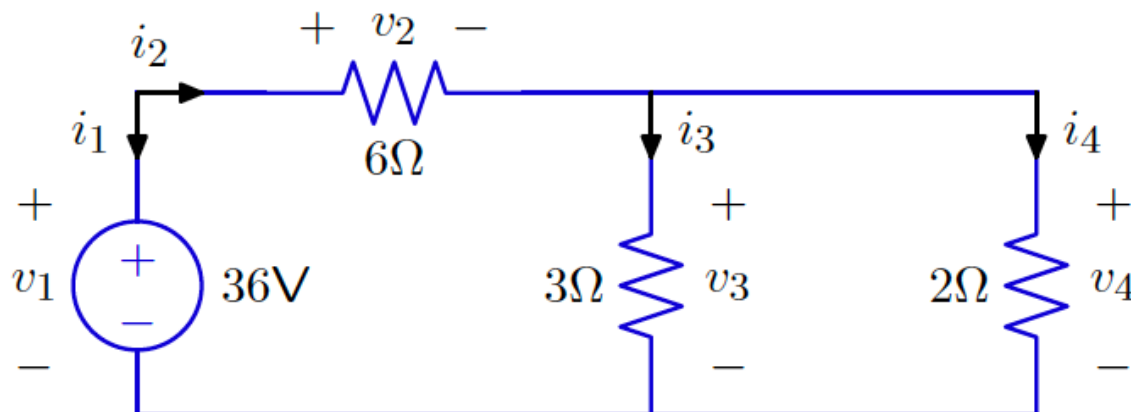
Find v_1 , v_2 , i_1 , and i_2 .



As before, $i_1 + i_2 = 0$ and $v_1 = v_2$. However, now we also know two constitutive relations: $v_1 = 10\text{V}$ and $v_2 = 2\Omega \times i_2$. Solving, we find that $v_1 = v_2 = 10\text{V}$, $i_2 = v_2/2\Omega = 5\text{A}$, and $i_1 = -i_2 = -5\text{A}$.

Example: Applying KVL, KCL, Ohm's

Solve the following circuit.



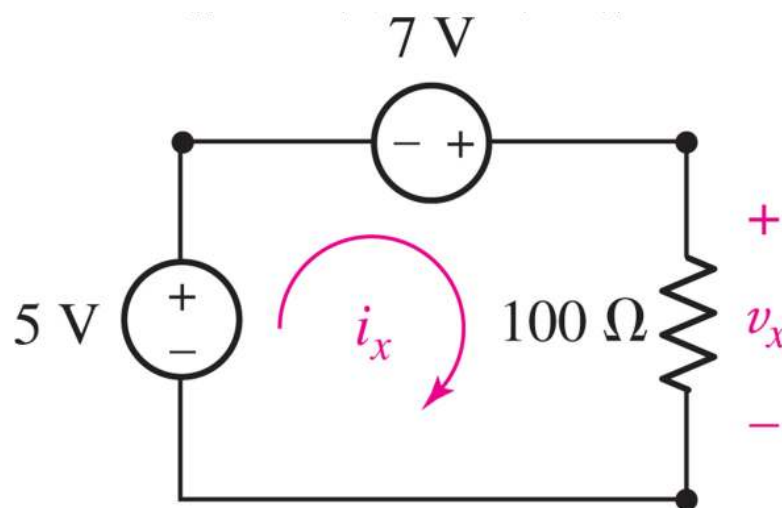
Answer:

$$i_1 = -5\text{A} ; \quad i_2 = 5\text{A} ; \quad i_3 = 2\text{A} ; \quad i_4 = 3\text{A} ;$$

$$v_1 = 36\text{V} ; \quad v_2 = 30\text{V} ; \quad v_3 = 6\text{V} ; \quad v_4 = 6\text{V}$$

Example: Applying KVL, KCL, Ohm's

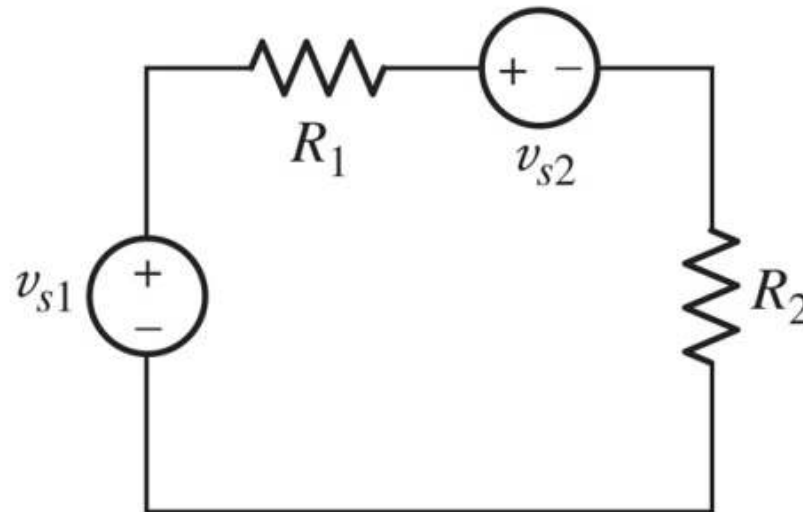
Example: find the current i_x and the voltage v_x



Answer: $v_x = 12\text{ V}$ and $i_x = 120\text{ mA}$

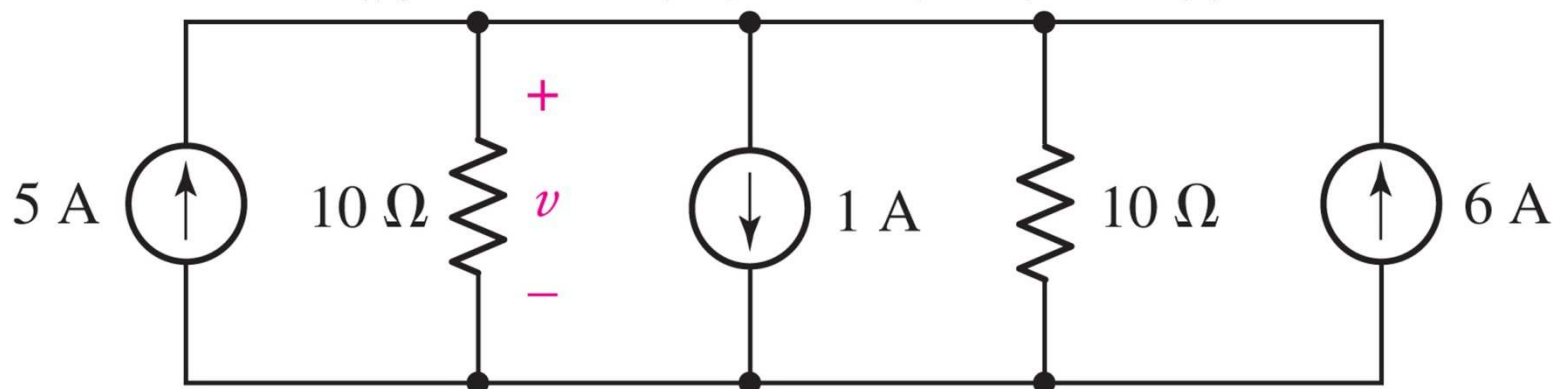
Series Connections

All of the elements in a circuit that carry the same current are said to be connected in **series**.



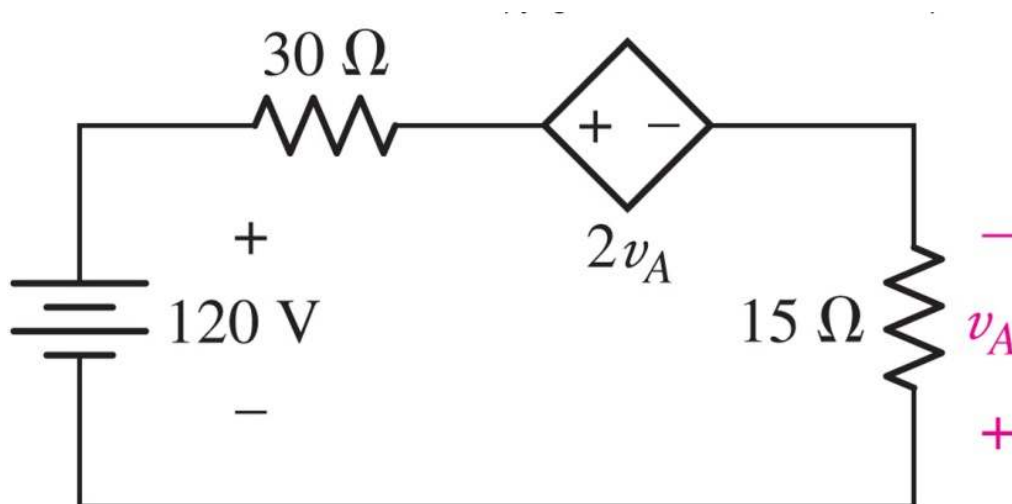
Parallel Connections

Elements in a circuit having a common voltage across them are said to be connected in **parallel**.



Example: Single Loop Circuit

Calculate the power absorbed by each circuit element.



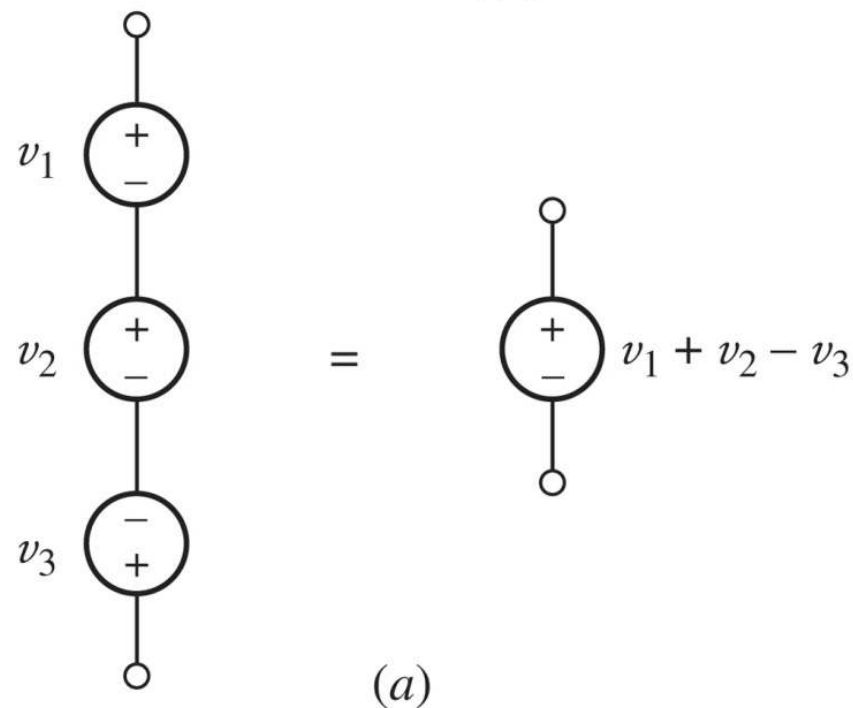
Answer:

$$p_{120V} = -960 \text{ W}, p_{30} = 1920 \text{ W}$$

$$p_{dep} = -1920 \text{ W}, p_{15} = 960 \text{ W}$$

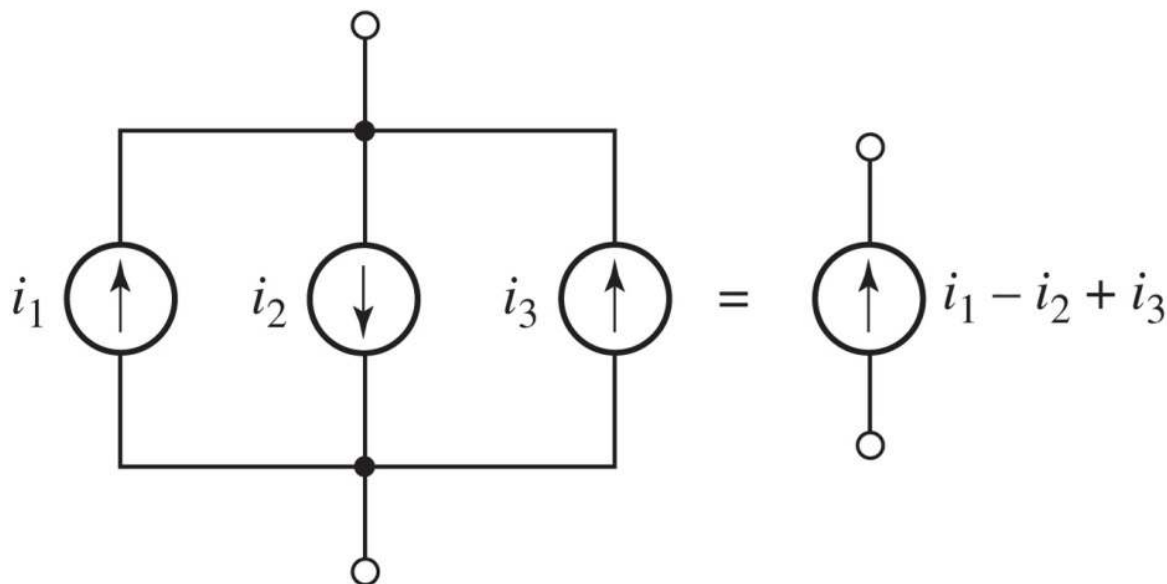
Series and Parallel Sources

Voltage sources connected in series can be combined into an equivalent voltage source:

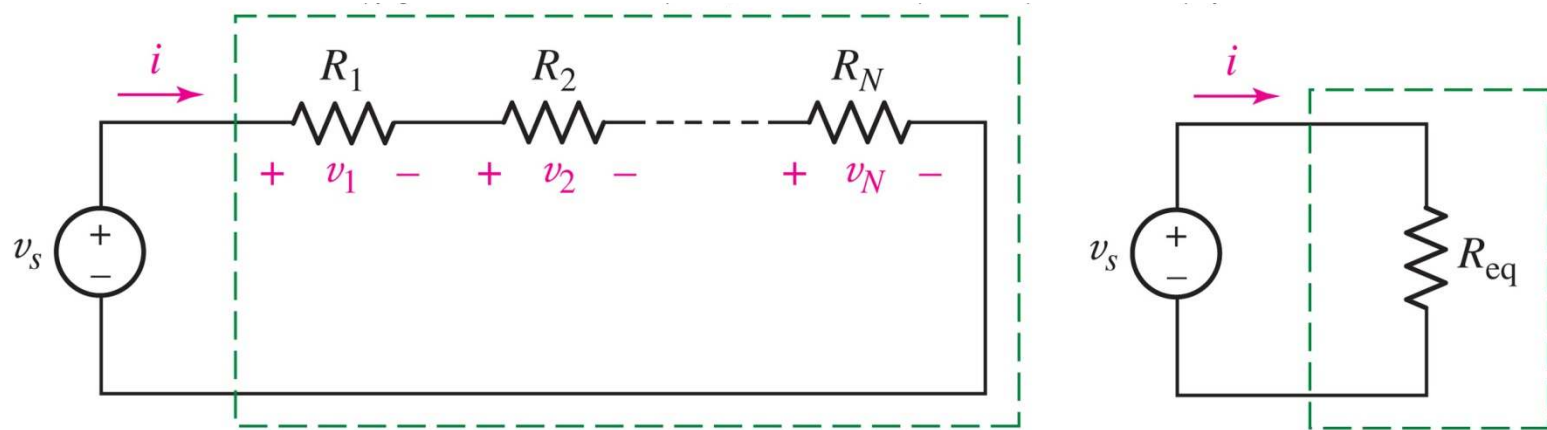


Series and Parallel Sources

Current sources connected in parallel can be combined into an equivalent current source:



Resistors in Series

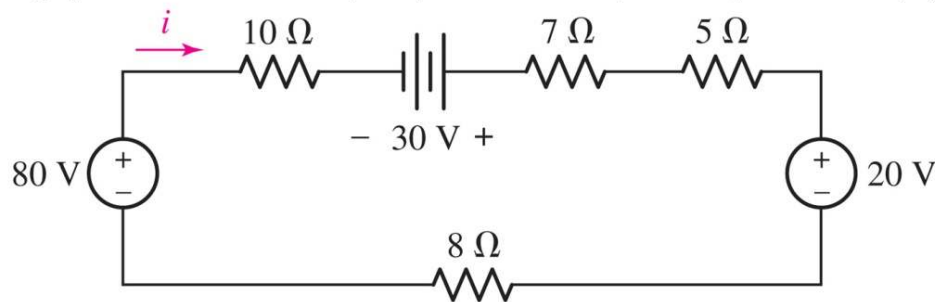
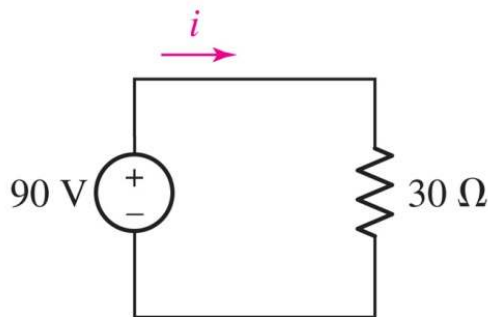
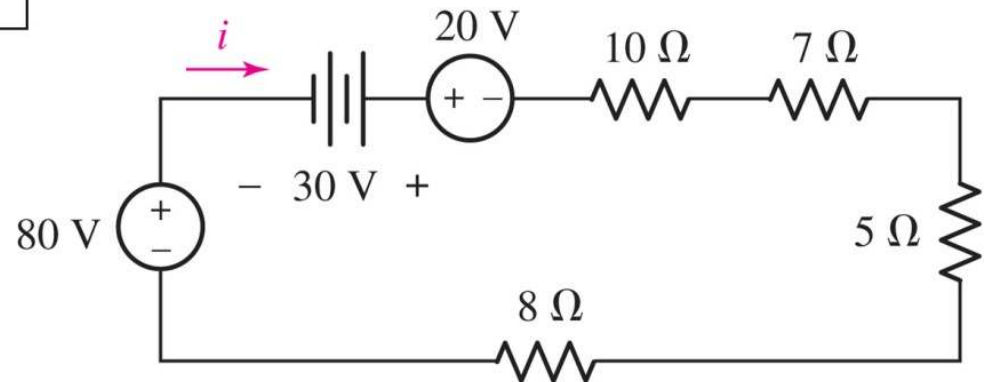


Using KVL shows:

$$R_{eq} = R_1 + R_2 + \dots + R_N$$

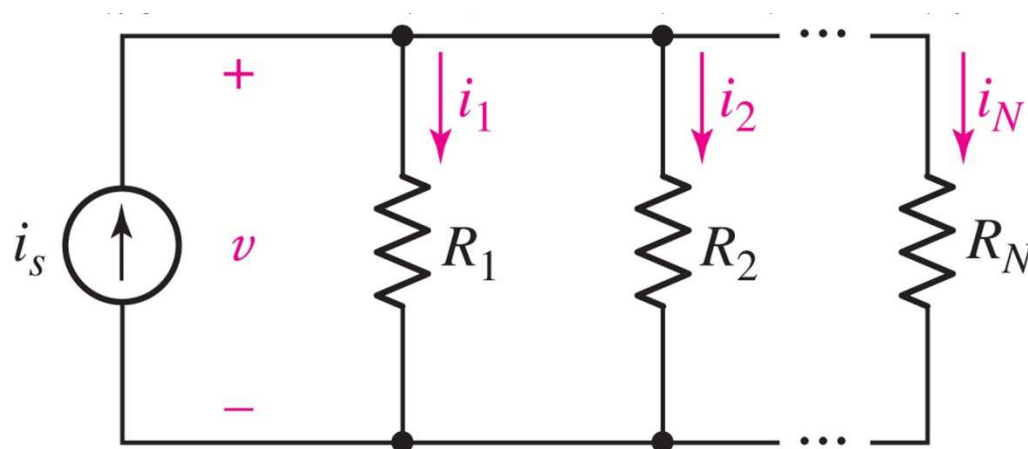
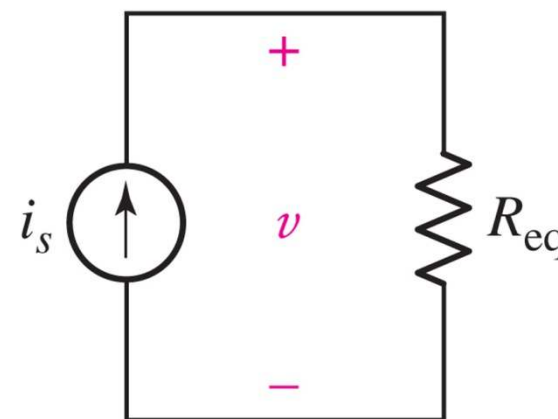
Example: Circuit Simplification

Find i and the power supplied by the 80 V source.



Answer: $i = 3\text{ A}$ and $p = 240\text{ W}$ supplied

Resistors in Parallel



Using KCL shows:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$$

Two Resistors in Parallel

$$R_{eq} = R_1 \parallel R_2$$
$$= \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

Two resistors in parallel can be combined using the **product / sum** shortcut.

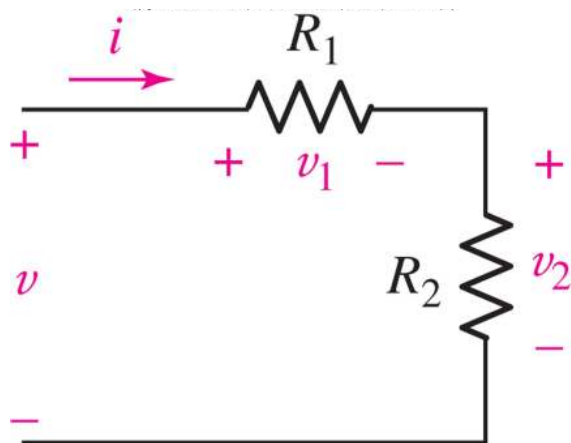
Connecting resistors in parallel makes the result *smaller* :

$$0.5 \min(R_1, R_2) < R_1 \parallel R_2 < \min(R_1, R_2)$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Voltage Division

Resistors in series “share” the voltage applied to them.

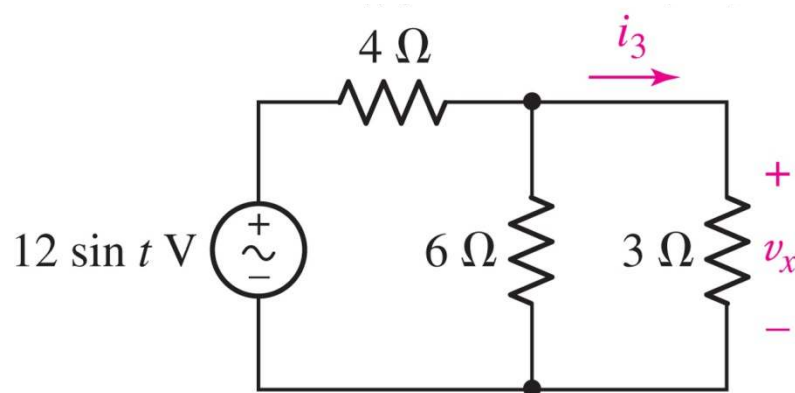
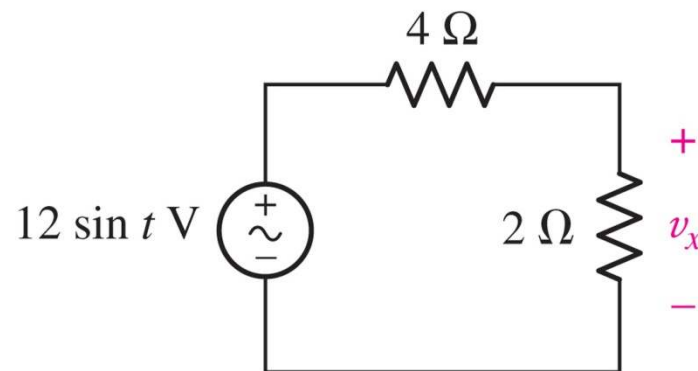


$$v_1 = \frac{R_1}{R_1 + R_2} v$$

$$v_2 = \frac{R_2}{R_1 + R_2} v$$

Example: Voltage Division

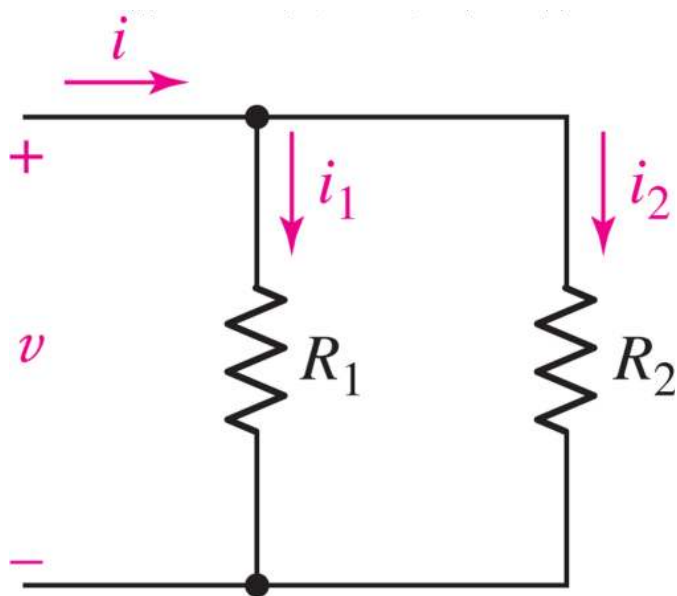
Find v_x



Answer: $v_x(t) = 4 \sin t \text{ V}$

Current Division

Resistors in parallel “share” the current through them.

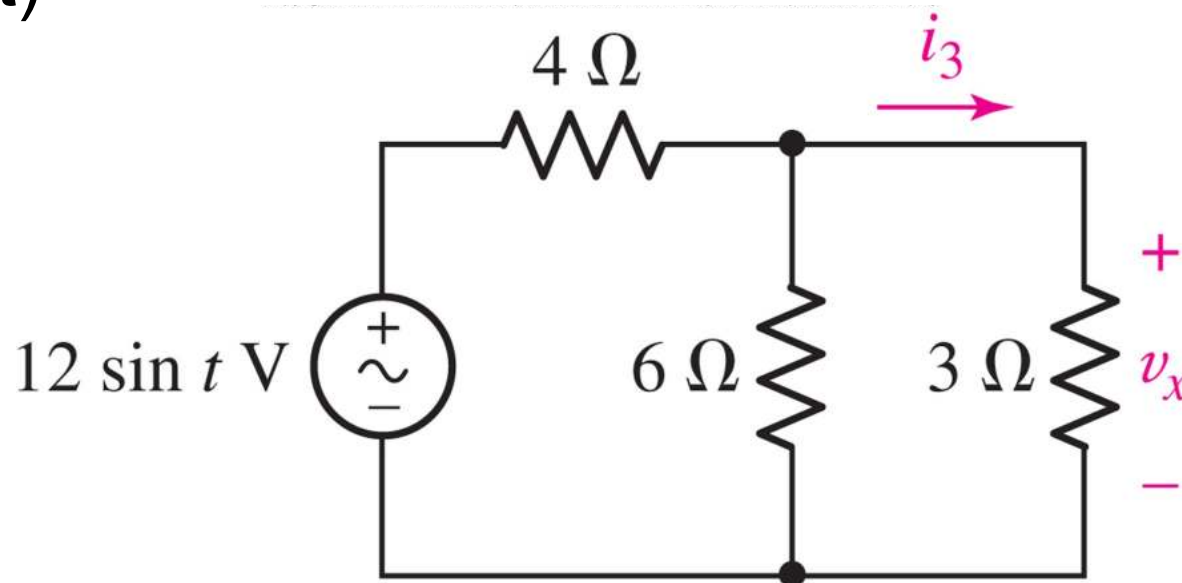


$$i_1 = i \frac{R_2}{R_1 + R_2}$$

$$i_2 = i \frac{R_1}{R_1 + R_2}$$

Example: Current Division

Find $i_3(t)$



Answer: $i_3(t) = 1.333 \sin(t) \text{ V}$

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