

Your Name:

Your Signature:

- **Exam duration:** 1 hour and 20 minutes.
- This exam is closed book, closed notes, closed laptops, closed phones, closed tablets, closed pretty much everything.
- **No calculators** of any kind are allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, **even if your answer is correct**.
- Place a box around your final answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- This exam has 13 pages, plus this cover sheet. Please make sure that your exam is complete, that you read all the exam directions and rules.

Question Number	Maximum Points	Your Score
1	20	
2	20	
3	25	
4	20	
5	15	
<i>Total</i>	100	

1. (20 total points) Find the Laplace transform or the inverse Laplace transform for the following functions. You may use the LT table.

(a) (5 points) $f_1(t) = 3e^{2t} \cos(11t) + 2e^{-3t} \sinh(7t)$. $F_1(s) = ?$

(b) (5 points) $F_2(s) = \frac{\sqrt{135}}{(s^2 + 3)^2}$. $f_2(t) = ?$

(c) (5 points) $f_3(t) = e^{2t}(t^3 + 5t - 2\cos(\frac{\pi t}{4}))$. $F_3(s)=?$

(d) (5 points) $F_4(s) = \frac{3s + 5}{s(s^2 + 2s + 5)}$. $f_4(t) = ?$ You'll have to solve this via partial fraction expansion.

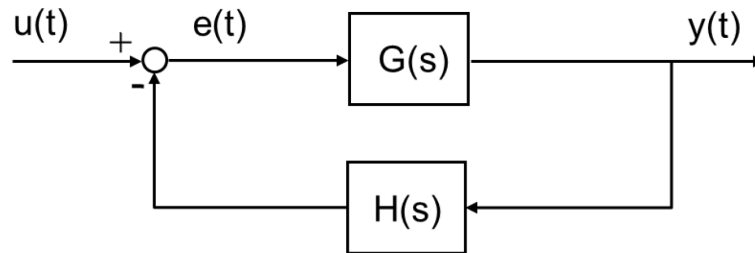
2. (20 total points) The following ODE is given:

$$y''(t) + 15y'(t) + 56y(t) = u(t)$$

(a) (20 points) Given that $y(0) = 0$ and $y'(0) = 0$, first compute $H(s) = \frac{Y(s)}{U(s)}$ then $y(t)$ if $u(t) = \pi \approx 3.14$.

3. (25 total points) For the system given in the below figure, assume that:

$$G(s) = \frac{1}{(s-1)(s+2)}, \quad H(s) = s+3.$$



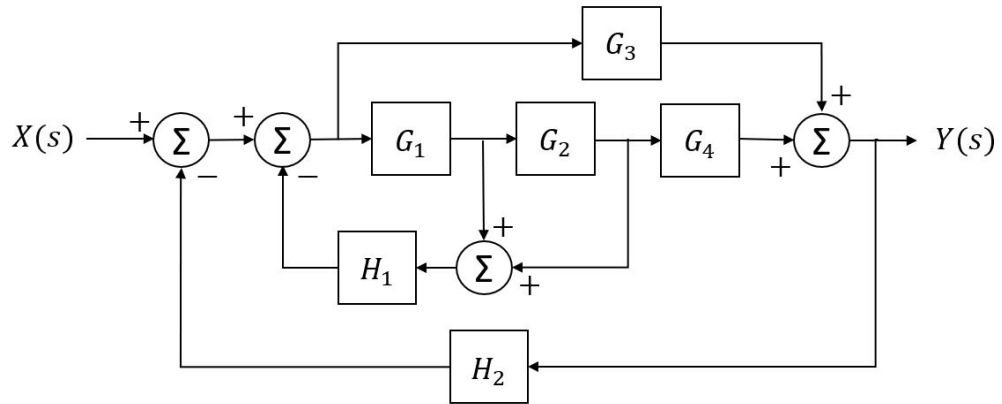
(a) (5 points) Find the transfer function $\frac{Y(s)}{U(s)}$. You can either derive it or just write it down immediately.

(b) (5 points) Find $Y(s)$ if $u(t) = 2$. DO NOT compute $y(t)$.

(c) (5 points) What are the poles of $Y(s)$? Does the final value of $y(t)$ exist (i.e., $y(\infty)$)? If it does, find it via the **final value theorem**. Otherwise, tell me why it doesn't.

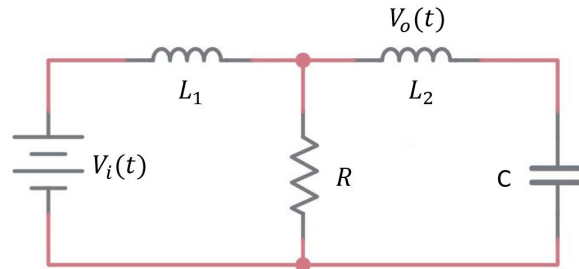
(d) (10 points) Obtain $\frac{E(s)}{U(s)}$, then find $E(s)$ for the given $u(t) = 2$. Does the final value of $e(t)$ exist (i.e., $e(\infty)$)? If it does, find it via the **final value theorem**. Otherwise, tell me why it doesn't.

4. (20 total points) You are given the following block diagram.



(a) (20 points) Find $\frac{Y(s)}{X(s)}$ for the above system. Show your work.

5. (15 total points) You are given the following RLC circuit.



(a) (15 points) Derive the transfer function $\frac{V_o(s)}{V_i(s)}$ in terms of R , L_1 , L_2 and C . Show your work. Your transfer function should have the standard form of a transfer function, i.e., polynomials in the numerator and denominator.

Table of Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}$	2. e^{at}	$\frac{1}{s-a}$
3. $t^n, n=1,2,3,\dots$	$\frac{n!}{s^{n+1}}$	4. $t^p, p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5. \sqrt{t}	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6. $t^{n-\frac{1}{2}}, n=1,2,3,\dots$	$\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7. $\sin(at)$	$\frac{a}{s^2+a^2}$	8. $\cos(at)$	$\frac{s}{s^2+a^2}$
9. $t \sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10. $t \cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11. $\sin(at) - at \cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12. $\sin(at) + at \cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13. $\cos(at) - at \sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14. $\cos(at) + at \sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15. $\sin(at+b)$	$\frac{s \sin(b) + a \cos(b)}{s^2+a^2}$	16. $\cos(at+b)$	$\frac{s \cos(b) - a \sin(b)}{s^2+a^2}$
17. $\sinh(at)$	$\frac{a}{s^2-a^2}$	18. $\cosh(at)$	$\frac{s}{s^2-a^2}$
19. $e^{at} \sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20. $e^{at} \cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21. $e^{at} \sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22. $e^{at} \cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23. $t^n e^{at}, n=1,2,3,\dots$	$\frac{n!}{(s-a)^{n+1}}$	24. $f(ct)$	$\frac{1}{c} F\left(\frac{s}{c}\right)$
25. $u_c(t) = u(t-c)$ <u>Heaviside Function</u>	$\frac{e^{-cs}}{s}$	26. $\delta(t-c)$ <u>Dirac Delta Function</u>	e^{-cs}
27. $u_c(t) f(t-c)$	$e^{-cs} F(s)$	28. $u_c(t) g(t)$	$e^{-cs} \mathcal{L}\{g(t+c)\}$
29. $e^{ct} f(t)$	$F(s-c)$	30. $t^n f(t), n=1,2,3,\dots$	$(-1)^n F^{(n)}(s)$
31. $\frac{1}{t} f(t)$	$\int_s^\infty F(u) du$	32. $\int_0^t f(v) dv$	$\frac{F(s)}{s}$
33. $\int_0^t f(t-\tau) g(\tau) d\tau$	$F(s)G(s)$	34. $f(t+T) = f(t)$	$\frac{\int_0^T e^{-st} f(t) dt}{1-e^{-sT}}$
35. $f'(t)$	$sF(s) - f(0)$	36. $f''(t)$	$s^2 F(s) - sf(0) - f'(0)$
37. $f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - sf^{(n-2)}(0) - f^{(n-1)}(0)$		