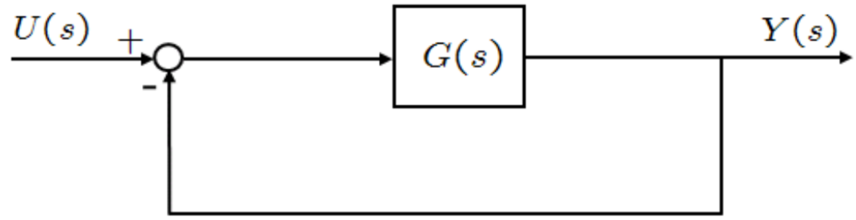
Your Name: | Your Signature:

- **Exam duration:** 2 hours.
- This exam is open book, open notes, open laptops, open phones, open tablets, open pretty much everything.
- You can use MATLAB whenever you want to find roots for polynomials, to compute angles, to approximate things, to verify your calculations, but your handwritten solutions need to be extremely detailed. I will be grading based on the level of detail in your solution.
- **No calculators** of any kind are allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, **even if your answer is correct.**
- Place a box around your final answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- This exam has 16 pages, plus this cover sheet. Please make sure that your exam is complete, that you read all the exam directions and rules.

Question Number	Maximum Points	Your Score
1	20	
2	30	
3	25	
4	25	
Total	100	
Bonus	15	



1. (20 total points) For the system shown in the above figure, assume that $G(s) = \dfrac{1}{s^2 + 4s + 20}$. Answer the following questions.

   (a) (5 points) What is the closed-loop transfer function (CLTF)?

   (b) (5 points) What are the poles of the CLTF?

(c) (5 points) Find the settling time of the CLTF via the 2% criterion, the maximum overshoot, and the peak time.

(d) (5 points) Compute the steady-state error (SSE) **corresponding to a unit ramp input**. Use whatever tables you want.

2. (30 total points) The characteristic polynomial (CP) of a system is given as follows:

$$1 + KG(s) = 1 + K\frac{1}{s^3 + 5s^2 + 24s + 20} = 1 + K\frac{1}{(s+1)(s^2+4s+20)}.$$

We want to sketch the root locus in this problem of the given CP.

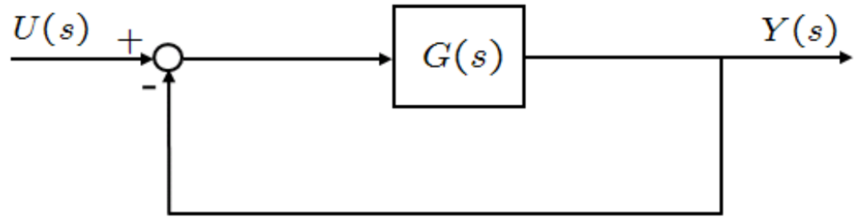
(a) (5 points) Find the poles, zeros of the OLTF. Find the part of the real axis on the root locus. How many branches does the root locus have?

(b) (5 points) Find the asymptotes of the root locus, including their angles (ϕ_q) and point of intersection (σ_a). The formulae are given on the last page of the exam.

(c) (10 points) Find the break-in/breakaway points of the root locus.

(d) (5 points) Find the angle of departure from the complex poles (you can only find one of these angles, since the RL is symmetric).

(e) (5 points) Find the crossings with the $j\omega$ axis, and sketch the root locus.



3. (25 total points) For the above system, consider that $G(s) = \frac{K}{s(s+10)^2}$, $K \geq 0$. For this problem, you can use the SSE table immediately.

   (a) (5 points) Via the Routh array, find the range of K such that the system is stable.

(b) (5 points) Given that $U_1(s) = \frac{1}{s}$, find the steady-state error e_{ss1} corresponding to $U_1(s)$ in terms of K.

(c) (5 points) Given that $U_2(s) = \frac{1}{s^2}$, find the steady-state error e_{ss2} corresponding to $U_2(s)$ in terms of K.

(d) (5 points) Given your solutions above, what is the overall SSE corresponding (e_{ss}) to an input $U(s) = 4U_1(s) + 5U_2(s)$? Your answer should also be in terms of K and you should use your solutions above (a,b).

(e) (5 points) For (d) above, we want the overall SSE to be equal to or less than 0.3 for the above given $U(s)$, i.e., $e_{ss} \leq 0.3$. Find the minimum value of K (i.e., K_{min}) that satisfies this requirement. Is this value possible?

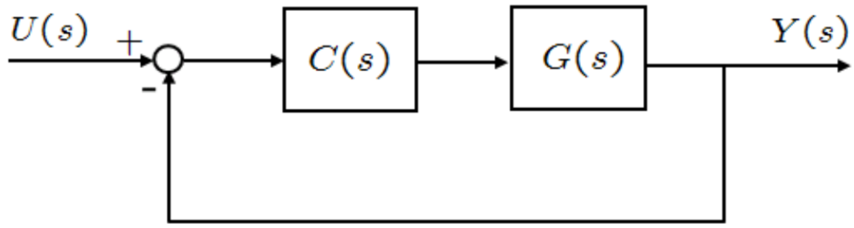
4. (25 total points) The characteristic polynomial of a closed-loop system is given by:

$$(2 + K)s^2 + (2 - 4K)s + 5K = 0.$$

   (a) (25 points) Plot the root locus. You should follow **all the steps we discussed in class**.

5. (15 total points) **[Bonus Question]** For the system shown in the below figure, assume that:

$$G(s) = \frac{1}{s^4 + 10s^3 + 20s^2 + 20s - 3}\ ,\ C(s) = 13 + \frac{K}{s}\ ,\ K \geq 0.$$



(a) (15 points) Find the CLTF, then using the Routh array criterion, find the range of K such that the CLTF is stable.