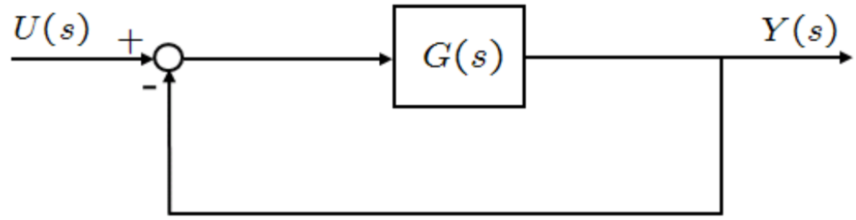
Your Name: Your Signature:

- **Exam duration:** 1 hour and 30 minutes.
- This exam is closed book, closed notes, closed laptops, closed phones, closed tablets, closed pretty much everything.
- **No calculators** of any kind are allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, **even if your answer is correct.**
- Place a box around your final answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- This exam has 16 pages, plus this cover sheet. Please make sure that your exam is complete, that you read all the exam directions and rules.
- **Question 6 is a bonus question. You do not need to answer it. You can, however, choose to answer it instead of another exam question. The maximum achievable grade is 110.**

Question Number	Maximum Points	Your Score
1	20	
2	20	
3	25	
4	15	
5	20	
Total	100	
Bonus	20	



1. (20 total points) For the system shown in the above figure, assume that $G(s) = \dfrac{1}{(s+2)^2}$. Answer the following questions.

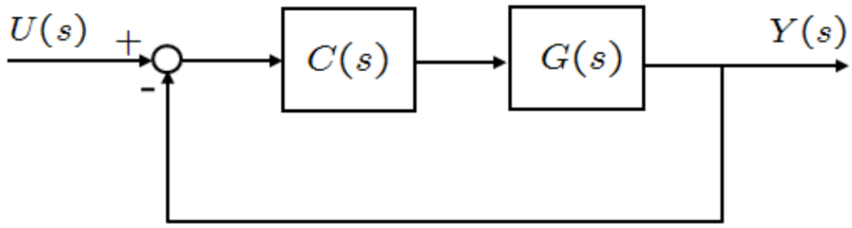
   (a) (2 points) What is the closed-loop transfer function (CLTF)?

   (b) (2 points) What are the poles of the CLTF?

(c) (3 points) Find the settling time (t_s) of the CLTF via the 5% criterion given that $t_s = \dfrac{3}{\zeta\omega_n}$.

(d) (3 points) Compute the steady-state error (SSE) corresponding to a **unit step input—you should use the provided SSE table.**

(e) (10 points) We want to design the system such that the settling time is $t_s = 1\,\text{sec}$ and damping ratio $\zeta = \frac{\sqrt{3}}{2}$. Find the corresponding desired CLTF poles given these design objectives.



2. (20 total points) For the system shown in the above figure, assume that:

$$G(s) = \frac{1}{s^3 + s^2 + 2s - 0.5}\,,\; C(s) = 1 + \frac{K}{s}\,,\; K \geq 0.$$

(a) (5 points) Find the CLTF.

(b) (5 points) Using the Routh array criterion, find the range of K such that the CLTF is **strictly stable**.

(c) (5 points) In terms of K, obtain the SSE of the above system given that the input is a unit ramp function. You'll have to figure out the **System Type** and use the **SSE table** (see last page of your exam booklet).

(d) (5 points) By varying the value of K, what is the smallest absolute value of such tracking error one can achieve (this part is related to (c) above)?

3. (25 total points) In this problem, we want to plot the root-locus for the following unity-feedback system with this open-loop transfer function (OLTF):

$$G(s) = K\frac{1}{(s^2+2s+2)(s-1)}.$$

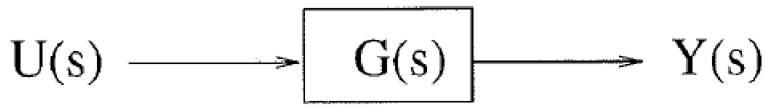
(a) (5 points) Find the poles, zeros (if any) of the OLTF. Find the part of the real axis on the root locus. How many branches does the root locus have?

(b) (5 points) Find the asymptotes of the root locus, including their angles (ϕ_q) and point of intersection (σ_a). The formulae are given on the last page of the exam.

(c) (5 points) Find the breakin/breakaway points of the root locus.

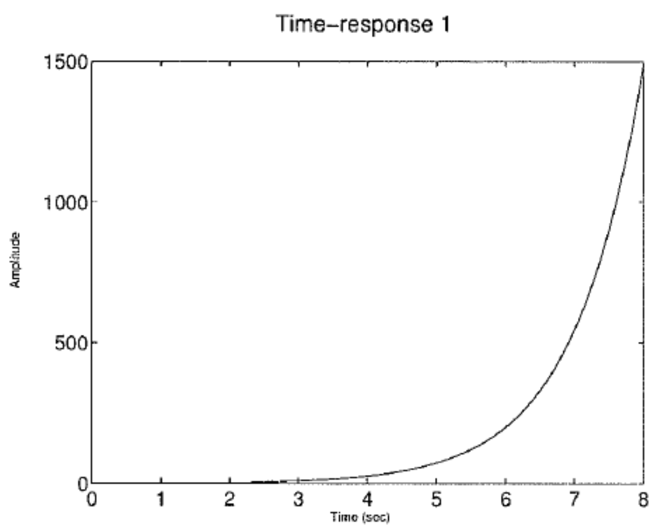
(d) (5 points) Find the angle of departure from the complex poles (you can only find one of these angles, since the RL is symmetric). You are also given that $\tan^{-1}(1/2) = 26.56\,\text{deg}$. The formulae are given on the last page of the exam.

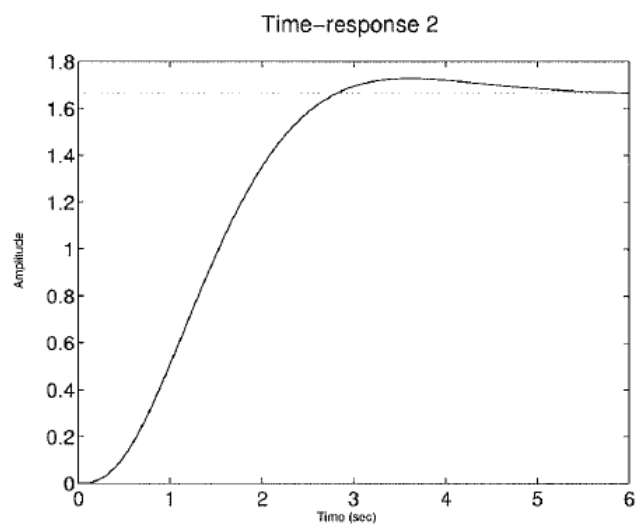
(e) (5 points) Find the crossings with the $j\omega$ axis, and sketch the root locus.

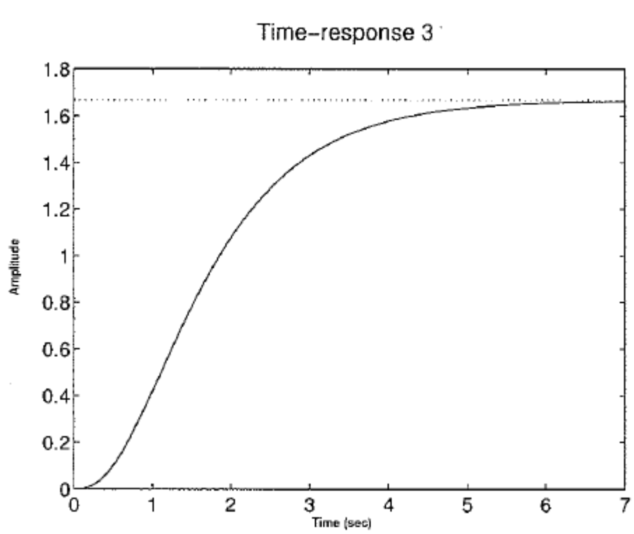


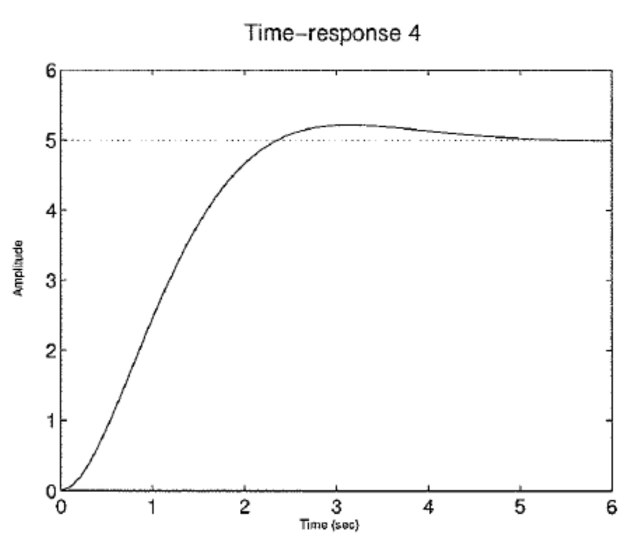
4. (15 total points) For the above system, match the four given step-response plots given in the below figure with the four transfer functions given in the below table. **Justify your answer in clear sentences. Lazy justifications get lazy credits.**

Transfer Function $G(s)$	$\frac{10}{(s+1)(s+2)(s+3)}$	$\frac{10}{(s^2+2s+2)(s+3)}$	$\frac{1}{(s-1)(s+1)}$	$\frac{10}{s^2+2s+2}$
Step Response Plot #	??	??	??	??

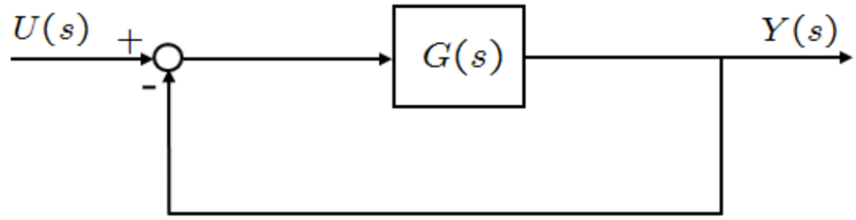








Your solution here:



5. (20 total points) For the above system, consider that

$$G(s) = \frac{K}{(s+3)(s+1)}, \; K \geq 0.$$

   (a) (5 points) Find the closed loop transfer function (CLTF).

   (b) (5 points) What are ω_n and ζ for this standard second order, CLTF in terms of K?

(c) (10 points) Find the value of K is the maximum overshoot (M_p) is equal to 0.1. The maximum overshoot is given by the following formula:

$$M_p = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi}.$$

You can use the following approximations: $\frac{\ln(0.1)}{\pi} \approx -0.75$, and $0.75^2 \approx 0.5$.

Your solution (K) can be approximate, but you have to clearly show all the steps involved.

6. (20 total points) The characteristic polynomial of a closed-loop system is given by:

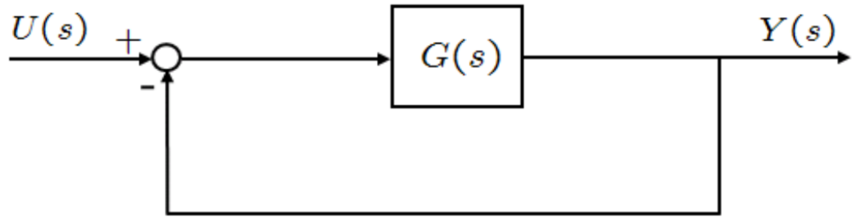
$$(1+K)s^2 + (2-2K)s + 2K = 0.$$

   (a) (20 points) Plot the root locus that corresponds to the above characteristic polynomial. You should follow **all the steps we discussed in class**. A simple sketch will not be graded. You are also given that $\tan^{-1}(1) = 45\,\text{deg}$ and $\tan^{-1}(1/3) = 18.43\,\text{deg}$.

   You should start by writing the above equation as $1 + KG(s)$ and then proceed with finding the poles and zeros of $G(s)$, the asymptotes (and their intersection and angles), breakaway/break-in points, angles of arrival/departure, and then a sketch. **For this problem, you do not need to compute the $j\omega$ axis crossing.**

   Helping hint: To solve for break-in/breakaway points, it is given that

$$\frac{dK}{ds} = 0 \Rightarrow (2s+2)(s^2 - 2s + 2) = 0.$$



SSE table for this system:

	Unit step input u(t)=1	Unit ramp input u(t)=t	Acceleration input u(t)=t²/2
Type 0 systems	$\frac{1}{1+K_p}$ $K_p = G(0)$	∞	∞
Type 1 systems	0	$\frac{1}{K_v}$ $K_v = \lim_{s\to 0} sG(s)$	∞
Type 2 systems	0	0	$\frac{1}{K_a}$ $K_a = \lim_{s\to 0} s^2G(s)$

Rule 6 Asymptotes angles: RL branches ending at OL zeros at ∞ approach the asymptotic lines with angles: $\phi_q = \dfrac{(1+2q)180}{n_p - n_z}$ deg, $\forall q = 0, 1, 2, \ldots, n_p - n_z - 1$

Rule 7 Real-axis intercept of asymptotes: $\sigma_A = \dfrac{\sum_{i=1}^{n_p} Re(p_i) - \sum_{j=1}^{n_z} Re(z_j)}{n_p - n_z}$

Rule 9 **Angle of Departure (AoD):** defined as the angle from a complex pole or Angle of Arrival (AoA) at a complex zero:

$$\text{AoD from a complex pole}: \ \phi_p = 180 - \sum_i \angle p_i + \sum_j \angle z_j,$$

$$\text{AoA at a complex zero}: \ \phi_z = 180 + \sum_i \angle p_i - \sum_j \angle z_j$$

- $\sum_i \angle p_i$ is the sum of all angles of vectors to a complex pole in question from **all other** poles, $\sum_j \angle z_j$ is the sum of all angles of vectors to a complex pole in question from **all other** zeros
- '$\angle$' denotes the angle of a complex number

Rule 10 Determine whether the RL crosses the imaginary y-axis by setting:

$$1 + KG(s = j\omega)H(s = j\omega) = 0 + 0i$$

and finding the ω and K that solves the above equation. The value of ω you get is the frequency at which the RL crosses the imaginary y-axis and the K you get is the associated gain for the controller. You should obtain two equations (real = 0 and imaginary = 0) with two unknowns (K,ω). From there, you solve for K,ω pairs