QUIZ # 1 SOLUTIONS

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A dynamical CTLTI system is characterized by $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 0.5 & 1 \end{bmatrix}$.

1. Find a linear state-observer gain $L = [l_1 \; l_2]^\top$ such that the poles of the estimation error are -5 and -7.

2. Can you place both poles at -6? If yes, what is the corresponding observer gain?

Solutions:

1. First, we find $A - LC$ in terms of l_1 and l_2:

$$A - LC = \begin{bmatrix} 1 - l_1/2 & 3 - l_1 \\ 3 - l_2/2 & 1 - l_2 \end{bmatrix}.$$

Since the roots of the designed observer are -5 and -7, the desired characteristic polynomial is:

$$\pi_{A-LC} = (\lambda + 5)(\lambda + 7) = \lambda^2 + 12\lambda + 35.$$

The characteristic polynomial in terms of l_1 and l_2 can be written as:

$$+\lambda^2 + \lambda \underbrace{\left(-2 + \frac{l_1}{2} + l_2\right)}_{=12} \underbrace{-8 + \frac{5l_1}{2} + \frac{l_2}{2}}_{=35} = 0.$$

Solving the following linear system of equations,

$$\begin{aligned} 35 &= -8 + \frac{5l_1}{2} + \frac{l_2}{2} \\ 12 &= -2 + \frac{l_1}{2} + l_2, \end{aligned}$$

we obtain $l_1 = 16$ and $l_2 = 6$.

2. Placing poles at $\lambda = -6$ means that

$$\pi_{A-LC} = (\lambda + 6)^2 = \lambda^2 + 12\lambda + 36$$

or,

$$\begin{aligned} 44 &= \frac{5l_1}{2} + \frac{l_2}{2} \\ 14 &= \frac{l_1}{2} + l_2, \end{aligned}$$

A solution to the above system of equations is $l_1 = 16.44$ and $l_2 = 5.77$.