HOMEWORK # 2 (VERSION 2)

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The objective of this homework is to test your understanding of the content of Module 2—Laplace transforms, transfer functions, partial fraction expansion, and ODE solutions. Provide neat solutions, with well-written answers. You need to have the Laplace transform table around you when you're doing the homework. Due date of the homework is: **Tuesday, February 4th at 1pm.**

You have to upload a scanned version of your solutions on Blackboard. If you don't have a scanner around you, you can use Cam Scanner—a mobile app that scans images in a neat way, as if they're scanned through a copier. Here's the link for Cam Scanner: https://www.camscanner.com/user/download.

1. Using Laplace transforms, solve the following differential equation for $y(t)$:

$$y'(t) - y(t) = e^{3t},$$

given that the initial value for $y(t)$ is $y(0) = 2$.

2. Using Laplace transforms, solve the following differential equation for $y(t)$:

$$y''(t) - 5y'(t) + 6y(t) = 5t,$$

given that $y(0) = -1$ and $y'(0) = 2$. Verify your answers on MATLAB via the `ilaplace` command.

3. For this differential equation:

$$y''(t) - 6y'(t) + 15y(t) = 2u(t),$$

solve the following problems:

(a) The transfer function $\dfrac{Y(s)}{U(s)}$.

(b) The poles and zeros (if any) of the transfer function.

(c) Given that $u(t) = \sin(3t), y(0) = -1, y'(0) = -4$, find $y(t)$ using partial fraction expansion. You might need to solve multiple linear equations with multiple unknowns. Do not panic.

(d) Verify your answers on MATLAB via the `ilaplace` command.

4. Prove that, using the definition of Laplace transform, the function $f(t) = 4e^{2t} + t$ is Laplace transformable and indicates the domain of $F(s)$. *Hint: find if there exists a value for s such that the Laplace integral is convergent for which* $0 < t < \infty$.

5. Find the Laplace transform of the given function:

$$f(t) = 3e^{2t}\cos(4t) + t\left(t^2 e^{-6t} - 2\sin(7t) + \tfrac{1}{t}\cos(-5t+1)\right).$$

6. Find the time domain solution for the given function:

$$f(t) = 3e^{2t} * (e^{-3t} - t),$$

where $*$ denotes the convolution operator. *Hint: use Laplace transform property for convolution.*

7. Using Laplace transforms, solve the following differential equation for $y(t)$:

$$y''(t) + 4y'(t) + 5y(t) = 3u(t),$$

given that initial conditions $y(0) = \frac{1}{4}$ and $y'(0) = 0$ with $u(t) = e^{-3t}$.