

The objective of this homework is to test your understanding of the content of Module 3. Due date of the homework is: **Tuesday, February 11th, 2020 at 1pm.**

You have to upload a scanned version of your solutions on Blackboard. If you don't have a scanner around you, you can use Cam Scanner—a mobile app that scans images in a neat way, as if they're scanned through a copier. Here's the link for Cam Scanner: <https://www.camscanner.com/user/download>.

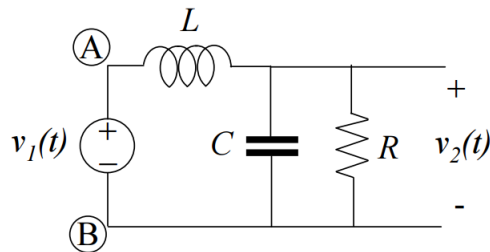
1. Linearize the following equation around $x_0 = \pi/2$:

$$y = f(x) = x^{1/3} + \cos(x).$$

Solution:

$$y_{linear} = f(x_0) + \left. \frac{\partial f(x)}{\partial x} \right|_{x=x_0} \cdot (x - x_0) = (\pi/2)^{1/3} + \left(\frac{1}{3}(\pi/2)^{-2/3} - 1 \right) (x - \pi/2)$$

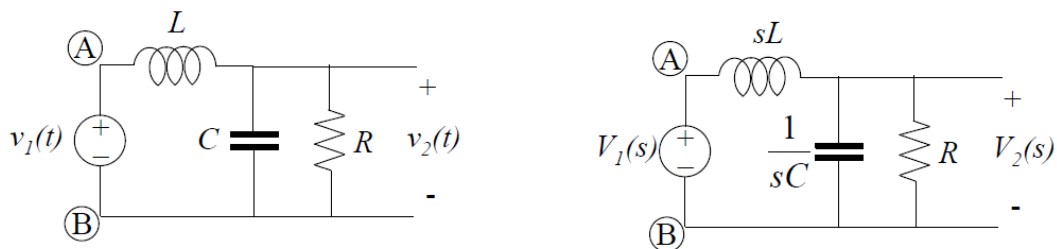
2. For this circuit:



find,

- (a) $\frac{V_2(s)}{V_1(s)}$ for any R, L, C .

Solution: The figure below shows the impedance illustration of the given circuit.



Given that, the equivalent impedance in this circuit is:

$$Z_{eq} = sL + R \parallel (1/sC) = \frac{RLCs^2 + Ls + R}{RCs + 1}.$$

Note that

$$\frac{V_2(s)}{V_1(s)} = \frac{Z_{R \parallel C}}{Z_{eq}} = \frac{R}{RLCs^2 + sL + R}.$$

- (b) $v_2(t)$ if $R = 1, L = 1, C = 1$, and

- i. $v_1(t) = \delta(t)$, or
- ii. $v_1(t) = 5$.

Solution: First, we substitute the value of the circuit elements in the transfer function obtained above; we get:

$$V_2(s) = \frac{V_1(s)}{s^2 + s + 1}.$$

For $v_1(t) = \delta(t)$, $V_1(s) = 1$, and hence:

$$v_2(t) = \mathcal{L}^{-1}\left[\frac{1}{s^2 + s + 1}\right] = \frac{2\sqrt{3}}{3}e^{-\frac{t}{2}} \sin(0.5\sqrt{3}t).$$

See Module 02 and the recitation session notes for more on that (or you can check the Laplace table). For $v_1(t) = 5$, $V_1(s) = 5/s$, and hence:

$$v_2(t) = \mathcal{L}^{-1}\left[\frac{1}{(s)(s^2 + s + 1)}\right] = 1 - 0.5 \cos(\sqrt{3}t) + \frac{\sqrt{3}}{3}e^{-\frac{t}{2}} \sin(0.5\sqrt{3}t).$$