THE UNIVERSITY OF TEXAS AT SAN ANTONIO
EE 3413
ANALYSIS AND DESIGN OF CONTROL SYSTEMS

HOMEWORK # 5
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The objective of this homework is to test your understanding of the content of Module 5. Due date of the homework is: **Thursday, March 5rd, 2020, at 1.00pm**.

You have to upload a scanned version of your solutions on Blackboard. If you don't have a scanner around you, you can use Cam Scanner—a mobile app that scans images in a neat way, as if they're scanned through a copier. Here's the link for Cam Scanner: https://www.camscanner.com/user/download.

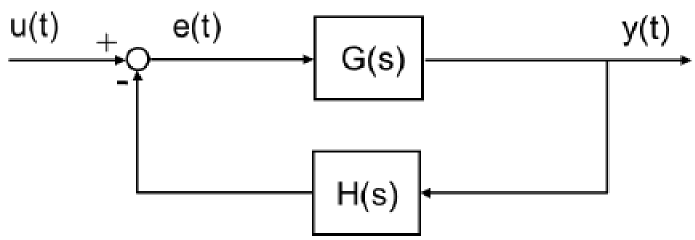

Figure 1: Feedback control system.

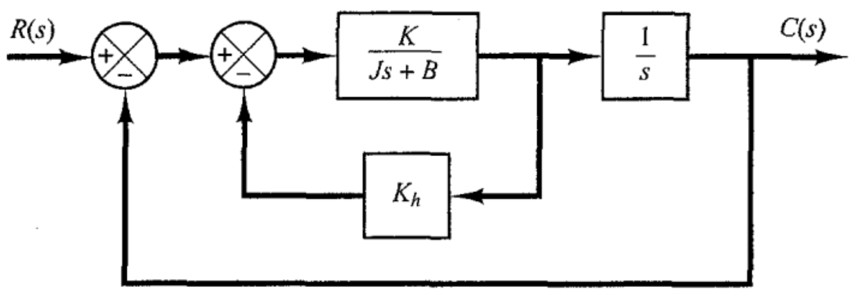

Figure 2: Servo system.

1. For a standard second order system given by this transfer function:

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2},$$

where $\zeta = 0.6$ and $\omega_n = 5$. Answer the following questions.

   (a) Find the: rise time, peak time, maximum overshoot, and settling time (the two criterion we discussed in class) if the system input is a unit step function.

   (b) Show a plot of how t_r, t_p, and M_p all vary with respect to different values of ζ and ω_n. Ideally, you should do that on MATLAB.

2. For the system shown in Figure 1, assume that $G(s) = \dfrac{-K}{s+10}$ and $H(s) = 1$. Answer the following questions:

   (a) Find the closed-loop transfer function $Y(s)/U(s)$ and its pole (or poles).

(b) What is the range of the constant K so that the closed-loop system is stable?

(c) Suppose K = 5. What is the time constant of the closed-loop transfer function (as a first order system)?

(d) What is the steady-state tracking error $e(\infty) = u(\infty) - y(\infty)$ under the input a unit step input $u(t)$?

3. For the system given in Figure 2, answer the following questions.

   (a) Obtain the transfer function $C(s)/R(s)$ in terms of constants K, J, B, K_h, and then write this system as a standard second order system as the transfer function given in Problem 1.

   (b) Determine the values of gain K and K_h so that M_p (the maximum overshoot) for a unit step response is equal to 0.2, and t_p (the peak time) is 1 second. Assume that $J = 1$ and $B = 1$.

   (c) With the above, now-obtained values for K and K_h, obtain the rise-time and settling time.

4. For the transfer functions below

$$T(s) = \frac{4}{s^2 + 2s + 4}$$

   find ζ, ω_n, settling time (2% and 5% of its final value), peak time, rise time, and percent overshoot.

5. Below is a mass-spring-damper system with $K = 28\,\frac{N}{m}$, $D = 5\,\frac{Ns}{m}$, and $M = 5\,kg$ (see Fig. 3)

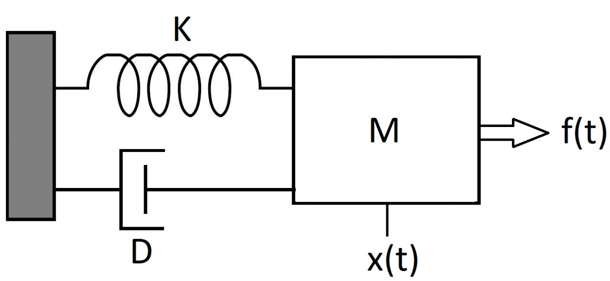

Figure 3: A mass-spring-damper system.

   Find its ζ, ω_n, settling time (2% and 5% of its final value), peak time, rise time, and percent overshoot where $x(t)$ is the center position of the mass and $f(t)$ is the force applied to the mass.

6. A system has a step response depicted in Fig. 4. Find its transfer function.

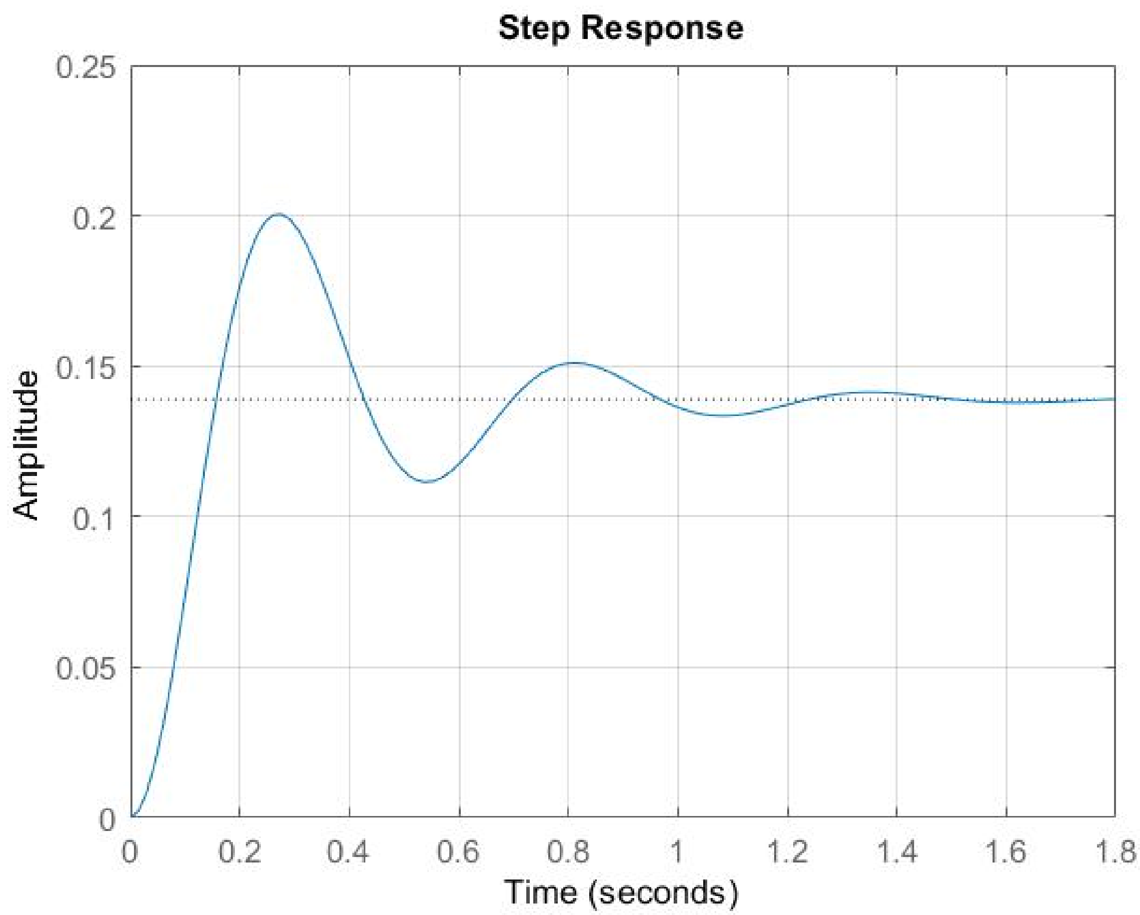

Figure 4: Step response.