THE UNIVERSITY OF TEXAS AT SAN ANTONIO
EE 3413
ANALYSIS AND DESIGN OF CONTROL SYSTEMS

HOMEWORK # 6
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The objective of this homework is to test your understanding of the content of Module 6. Due date of the homework is: **Tuesday, March 17th, 2020, @ 1pm**.

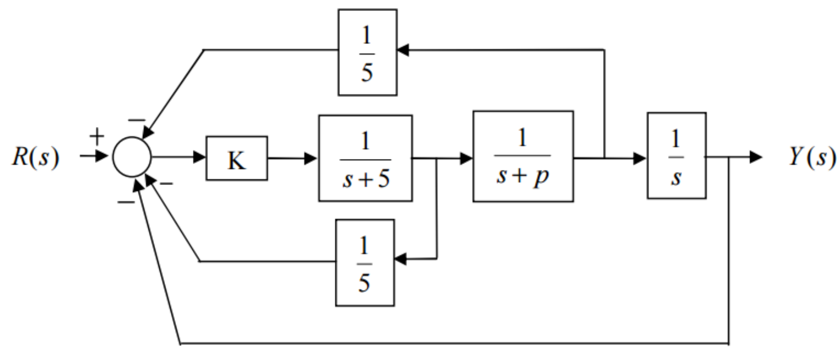

Figure 1: Spark ignition system — a block diagram representation.

1. Using the Routh-Array method we discussed in Module 6 (in addition to the two special cases we discussed), determine whether the following system has any poles in the right half plane. Determine the number of these RHP poles.

$$H(s) = \frac{1}{s^4 + s^3 + 12s^2 + 12s + 36}.$$

2. For this CLTF,

$$H(s) = \frac{1}{s^3 + 7s^2 + 11s + (5+K)},$$

determine the range of K (which could include negative numbers, for this problem only) such that the system has no poles in the RHP. Consider all the cases.

3. The system shown in Figure 1 is a typical block diagram representation depicting a simplified spark ignition system of an engine. Two parameters of this system are the gain K and performance parameter p. The performance parameter can take two values: $p_1 = 0$ and $p_2 = 2$. The objective of this problem is to design a gain K to stabilize the initially unstable system. Answer the following questions.

   (a) Find the overall closed loop transfer function $\frac{Y(s)}{R(s)}$ in terms of p and K. You should end up with a typical TF with polynomials on the denominator and numerator. Your CLTF should be third order TF. Make sure that your answer is correct before you move to the next question.

   (b) Obtain a value (or values) for K that would make the CLTF stable for the two given values of p, **simultaneously**. In other words, your design should stabilize the system whether $p = 0$ or $p = 2$.
   Hint: you should end up with two Routh-Arrays in terms of p and K.

4. For the unity feedback system in shown in Figure 2, the open-loop TF is given as follows:

$$G(s) = \frac{K(s+\alpha)}{s(s+\beta)},$$

where K, α and β are parameters that I want you to design. The design objectives are:

- Steady-state error $\frac{1}{10}$;
- Closed-loop poles are: $p_{1,2} = -1 \pm j$.

Find α, β and K such that the above design objectives are satisfied.

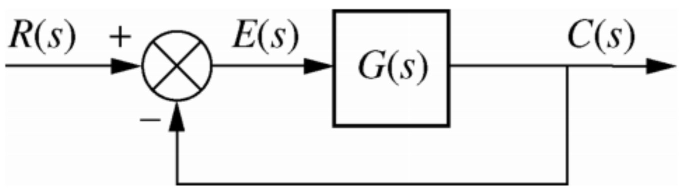

Figure 2: Unity feedback system.

5. Determine the stability of the following systems with unity feedback shown in Figure 2 via Routh-Hurwitz method:

   (a)
   $$G(s) = \frac{240}{(s+4)(s+3)(s+2)(s+1)}$$

   (b)
   $$G(s) = \frac{1}{2s^4 + 5s^3 + s^2 + 2s}$$

6. Find the range of η such that the closed-loop systems of the following transfer functions are stable given the unity feedback shown in Figure 2:

   (a)
   $$G(s) = \frac{(s+2)\eta}{s(s+3)(s-1)}$$

   (b)
   $$G(s) = \frac{(s+5)(s+3)\eta}{(s-4)(s-2)}$$

   (c)
   $$G(s) = \frac{\eta}{(s+2+j)(s+2-j)(s+7j)(s-7j)}$$