HOMEWORK # 7 SOLUTIONS

Ahmad F. Taha

April 10, 2020

Plot the root-locus for the following **unity-feedback** systems. You should apply the 10 Rules we discussed in class; you should find breakaway/break-in points, angle of departures, asymptotes, $j\omega$-axis crossings, and range of K such that the system is stable. You should also verify your solutions via MATLAB.

1. $G(s) = K\dfrac{s^2+4s+8}{s(s-2)}$

   (a) Poles: $p_{1,2} = 0, +2$. Zeros: $z_{1,2} = -2 \pm 2j$. $n_p = n_z = 2$

   (b) Asymptotes: none as $n_p - n_z \leq 0$

   (c) Breakaway points:

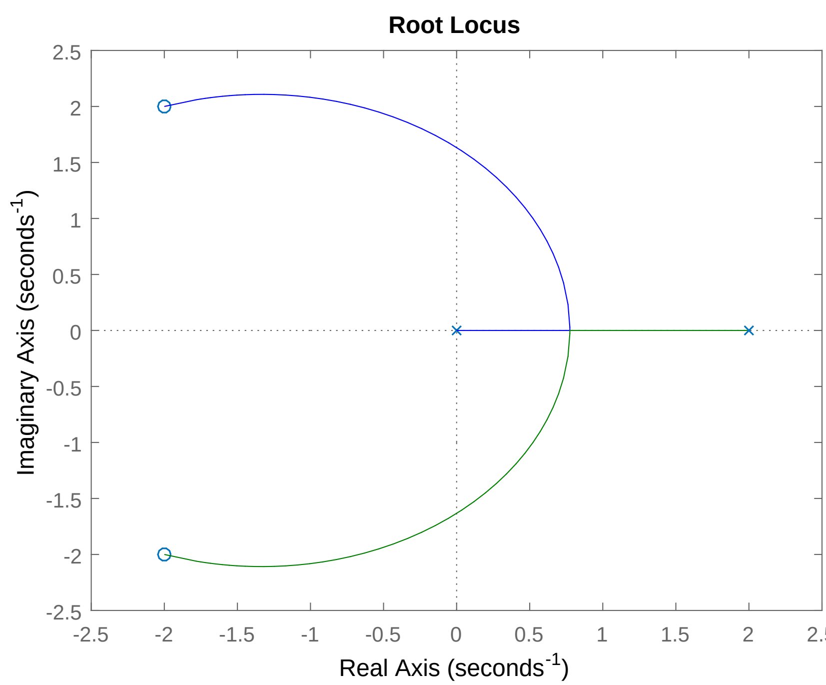
   $$\frac{dK}{ds} = 0 \Rightarrow 3s^2 + 8s - 8 = 0 \Rightarrow s_{1,2} = -3.44, 0.775 \Rightarrow s_2 = 0.775 \text{ is a breakaway point}$$

   (d) Angles of arrival at the complex zeros:

   $$\phi_{z_1} = 180 - 90 + (180 - \arctan(2/2)) + (180 - \arctan(2/4)) = 18.43 \text{ deg} \Rightarrow \phi_{z_2} = -18.43 \text{ deg}$$

   (e) $j\omega$ axis crossing: $K \approx 0.475, \omega \approx \pm 1.6$

   (f) Plot:



2. $G(s) = K\frac{1}{(s^2+2s+2)(s^2+2s+5)}$

(a) Poles: $p_{1,2,3,4} = -1 \pm j, -1 \pm 2j$. Zeros: None $n_p = 4, n_z = 0$

(b) Asymptotes: $\phi_q = 45, -45, 135, -135$ degrees, $\sigma_a = (-1-1-1-1)/4 = -1$

(c) Breakaway points:

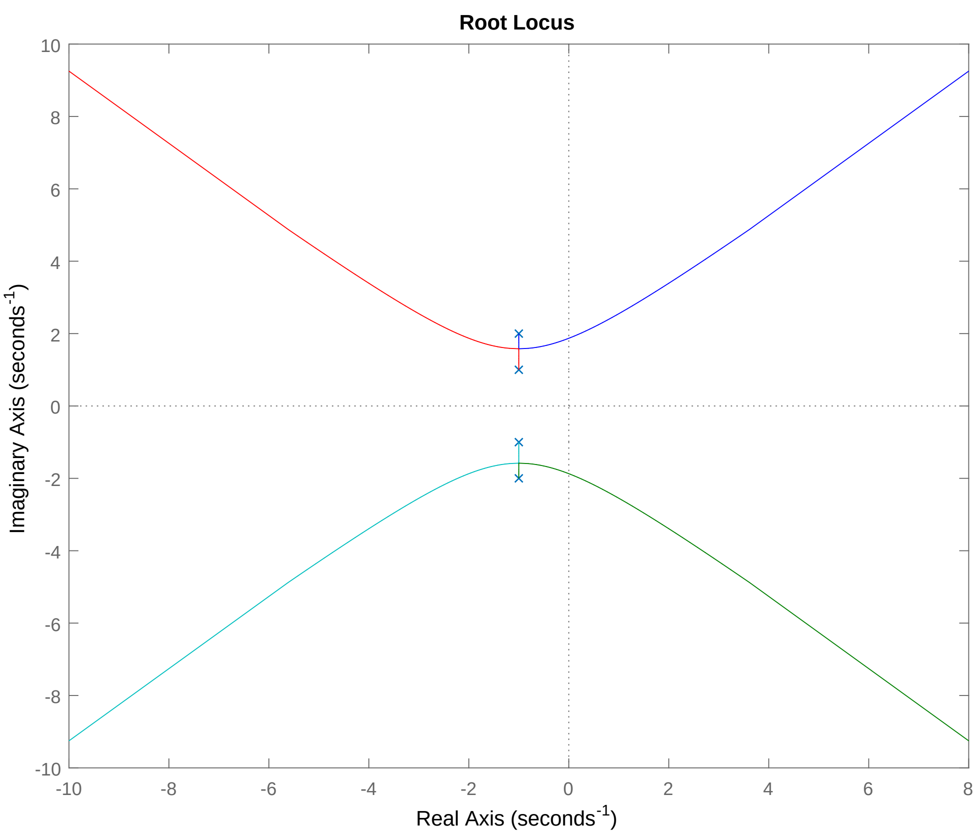
$$\frac{dK}{ds} = 0 \Rightarrow -(2s+2)(2s^2+4s+7) = 0 \Rightarrow s_{1,2,3} = -1, -1 \pm 1.58j \Rightarrow s_{2,3} \text{ are the complex breakaway point}$$

(d) Angles of departure at the complex poles:

$$\phi_{p_{1,2,3,4}} = -90, 90, 90, -90$$

(e) $j\omega$ axis crossing: $K \approx 16.25, \omega \approx \pm 1.87$

(f) Plot:



3. $G(s) = K\dfrac{(s+1)}{s(s+2)(s+3)(s+5)}$

(a) Poles: $p_{1,2,3,4} = 0, -2, -3, -5$. Zeros: $z_1 = -1$. $n_p = 4, n_z = 1$

(b) Asymptotes: $\phi_q = 60, 180, -60$ degrees, $\sigma_a = (-2-3-5+1)/3 = -3$

(c) Breakaway points:

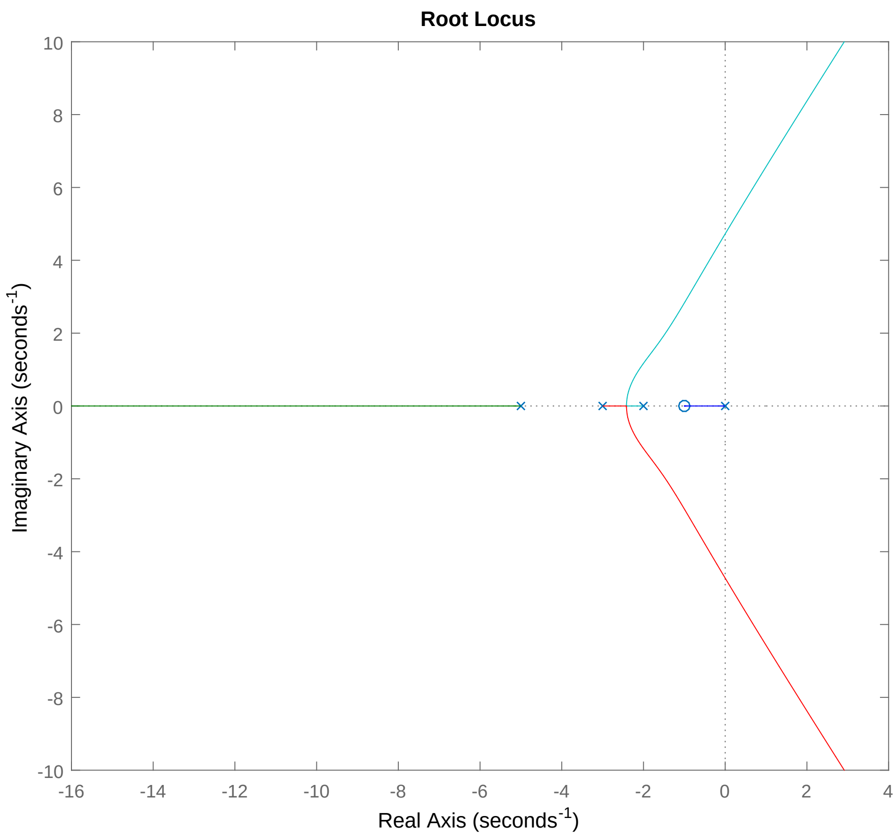
$$\frac{dK}{ds} = 0 \Rightarrow 3s^4 + 24s^3 + 61s^2 + 62s + 30 = 0$$

$\Rightarrow s_{1,2,3,4} = -4.18, -2.4174, -0.6989 \pm 0.7071j \Rightarrow s_2 = -2.417$ is the breakaway point

(d) Angles of departure at the complex poles: None

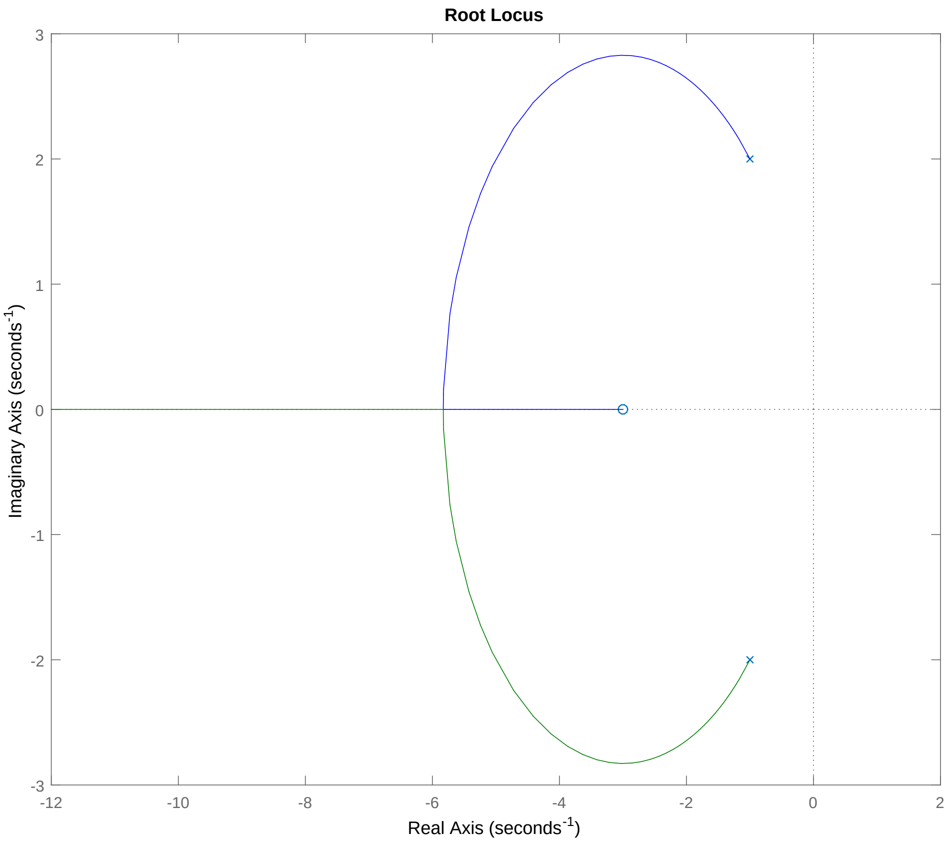
(e) $j\omega$ axis crossing: $K \approx 193, \omega \approx \pm 4.72$

(f) Plot:



4. $G(s) = K\dfrac{s+3}{s^2+2s+5}$.

Solution similar to one of the examples we did in class. Here's the plot:



5. Draw the Root-Locus given that this is the characteristic polynomial of the closed-loop system:

$$(1+K)s^2 + (2-2K)s + 2K = 0.$$

Hint: *for Problem 5, you should write the polynomial as* $1 + KG(s) = 0$, *and then follow the typical steps to draw the root-locus.*

(0) Using the hint, we can write the given characteristic polynomial as a standard $1 + KG(s) = 0$:

$$(1+K)s^2 + (2-2K)s + 2K = 0 \Rightarrow s^2 + 2s + K(s^2 - 2s + 2) = 0 \Rightarrow 1 + K\frac{s^2 - 2s + 2}{s^2 + 2s} = 0.$$

Hence, the new $G(s)$, is equal to $\tilde{G}(s) = \dfrac{s^2 - 2s + 2}{s^2 + 2s}$.

(a) Poles: $p_{1,2} = 0, -2$. Zeros: $z_{1,2} = 1 \pm j$. $n_p = 2, n_z = 2$

(b) Asymptotes: None, since $n_p - n_z \leq 0$

(c) Breakaway points:

$$\frac{dK}{ds} = 0 \Rightarrow (2s+2)(s^2 - 2s + 2) = 0$$

$$\Rightarrow s_{1,2,3} = -1, -0.62, 1.62 \Rightarrow s_2 = -0.62 \text{ is the breakaway point}$$

(d) Angles of arrival at the complex zeros:

$$\phi_{z_1} = 180 + 45 + \arctan(1/3) - 90 = 153.43\,\text{deg} \Rightarrow \phi_{z_2} = -153.43\,\text{deg}$$

(e) $j\omega$ axis crossing: $K \approx 0.95, \omega \approx \pm 1$

(f) Plot: