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EE 3413
4/14/16
HW #8

1) First solution

$G(s) = \frac{10}{s(s+1)}$

$\zeta_d = 0.5$

$\omega_{nd} = 3$

$s_d = -\zeta_d \omega_{nd} \pm \sqrt{1-\zeta_d^2} \cdot \omega_{nd}$

$= (-0.5)(3) \pm \sqrt{1-0.25}\,(3)$

$= \frac{-3}{2} \pm \frac{\sqrt{3}}{\sqrt{4}}(3)$

$= -\frac{3}{2} \pm \frac{3}{2}\sqrt{3}$

$S_d = -1.5 \pm j2.59$

ϕ angle of dificiency

$\theta = \angle G(s_d) = \angle G(-1.5+j2.59) = 138$

$\phi = -180-(138) = -318 = 42$ deg

$G_c^{ld}(s) = K \frac{s+z}{s+p}$

$z = 1.9432$

$p = 4.6458$

Characteristic

$1 = |K G(s) \cdot C(s)|$

$1 = \left| K \frac{10}{s(s+1)} \cdot \frac{s+1.9432}{s+4.6458} \right|$

$K = \left| \frac{s(s+1)(s+4.6458)}{10(s+1.9432)} \right|$

$s = -1.5 \pm j2.59$

$= \frac{(-1.5+j2.59)(-.5+j2.59)(3.1458+j2.59)}{4.432+25.9j}$

$= \left| \frac{-5.3748-31.935j}{4.432+25.9j} \right| \Rightarrow K = 1.213$

$G_c^{ld}(s) = K_c \frac{s+z}{s+p}$

$K_c = 1.213$

$z = 1.9432$

$p = 4.6458$

CLTF: $\frac{G_c(s) \cdot G(s)}{1+G_c(s)G(s)H(s)}$

$= \frac{\left(1.213 \frac{s+1.9432}{s+4.6458}\right)\left(\frac{10}{s(s+1)}\right)}{1+1.213\left(\frac{s+1.9432}{s+4.6458}\right)\left(\frac{10}{s(s+1)}\right)}$

$= \frac{1.213(s+1.9432)(10)}{(s+4.6458)s(s+1)+1.213(s+1.9432)(10)}$

$= \frac{12.13s+23.57}{s^3+5.6458s^2+16.7758s+23.57}$

STATIC-VELOCITY ERROR CONSTANT:

$K_v = \lim_{s\to 0} s\, G_c(s) G(s)$

$= \lim_{s\to 0} s\left[1.213 \cdot \frac{s+1.9432}{s+4.6458} \cdot \frac{10}{s(s+1)}\right]$

$= 5.07$

1) second solution

$$G_c(s) = K_c \frac{s+1}{s+3}$$

Characteristic

$$\left| K_c \frac{\cancel{s+1}}{s+3} \frac{10}{s\cancel{(s+1)}} \right| = 1$$

$$K_c \frac{10}{(s+3)s} = 1$$

$$K_c = \frac{s(s+3)}{10}$$

$$s = -1.5 + j\,2.5981$$

$$= \frac{(-1.5+j2.5981)(1.5+j2.5981)}{10}$$

$$K_c = \left| \frac{-2.25-6.750}{10} \right| = |-.9| = 0.9$$

$$G_c^{ld}(s) = K_c \frac{s+z}{s+p}$$

$$= 0.9 \frac{s+1}{s+3}$$

$$CLTF = \frac{G_c(s)G(s)}{1+G_c(s)G(s)H(s)}$$

$$= \frac{0.9 \frac{s+1}{s+3} \frac{10}{s(s+1)}}{1+0.9 \frac{s+1}{s+3} \frac{10}{s(s+1)}}$$

$$= \frac{9}{s(s+3)+9}$$

$$= \frac{9}{s^2+3s+9}$$

Static-Velocity Error Constant

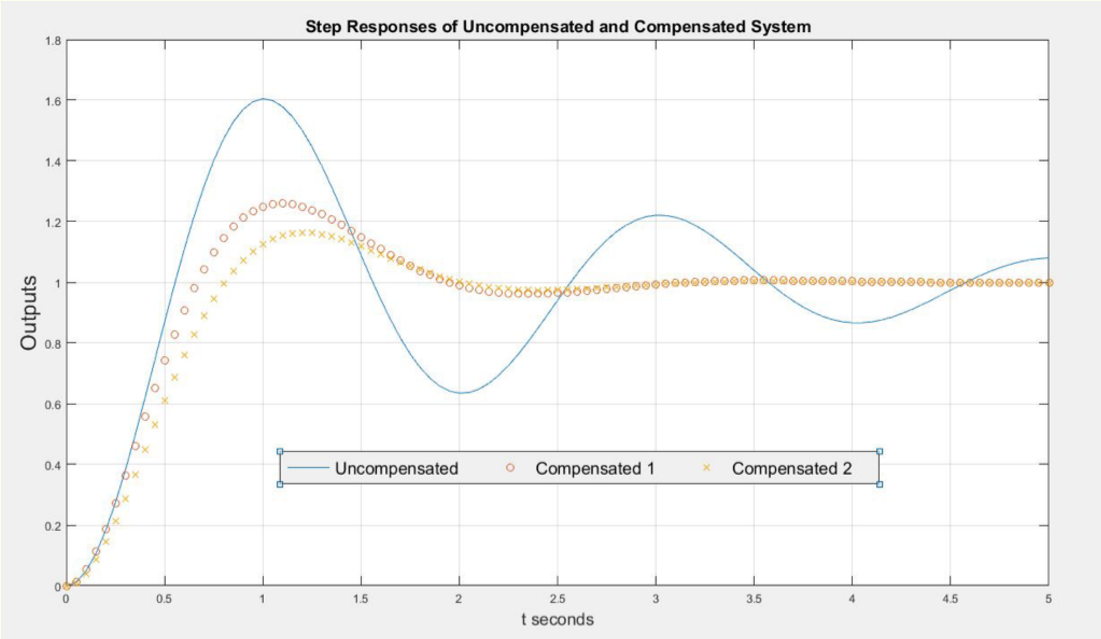
$$K_v = \lim_{s \to 0} s\left(G_c(s)G(s)\right)$$

$$= \lim_{s \to 0} s\left[0.9 \frac{\cancel{s+1}}{s+3} \frac{10}{s\cancel{(s+1)}}\right]$$

$$= \lim_{s \to 0} \cancel{s}\left[\frac{9}{(s+3)\cancel{s}}\right]$$

$$= \frac{9}{0+3} = 3$$

1)



Compensated 1 shows a faster rise time and overshoot but a slower settling time compared to compensator 2.

Compensated 2 shows a quicker settling time but a slower rise time and overshoot compared to compensated 1.

2) $G_c(s) = K(Ts+1)$

$G(s) = \dfrac{1}{s(s+2)}$

loop system poles:

$-2 \pm j2$

standard form:

$\dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$

CLTF: $\dfrac{G_c(s)G(s)}{1 + G_c(s)G(s)H(s)}$

$= \dfrac{K(Ts+1)\left(\frac{1}{s(s+2)}\right)}{1 + K(Ts+1)\left(\frac{1}{s(s+2)}\right)}$

$= \dfrac{K(Ts+1)}{s(s+2) + K(Ts+1)}$

$= \dfrac{KTs + K}{s^2 + (2s + KTs) + K}$

$= \dfrac{-(2+KT) \pm \sqrt{(2+KT)^2 - 4(1)(K)}}{2}$

$= \dfrac{-(2+KT)}{2} = -2$

$= 2 + KT = 4$

$KT = 2$

$\dfrac{\sqrt{(2+KT)^2 - 4k}}{2} = 2j$

$\left(\dfrac{\sqrt{(2+KT)^2 - 4k}}{2}\right)^2 = (2j)^2$

$\dfrac{(2+2)^2 - 4K}{4} = -4$

$16 - 4k = -16$

$k = 8$

$T = \dfrac{2}{8} = \dfrac{1}{4}$

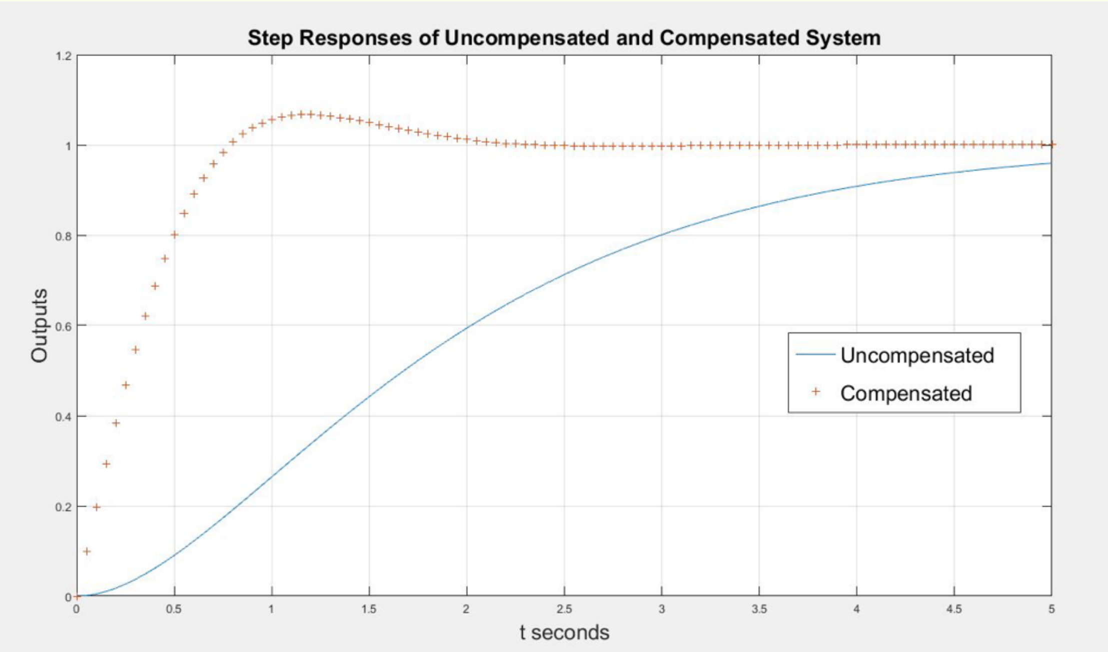
CLTF: $\dfrac{KTs + K}{s^2 + (2+KT)s + K}$

$= \dfrac{8\frac{1}{4}s + 8}{s^2 + (2 + 8\frac{1}{4})s + 8} \Rightarrow \dfrac{2s + 8}{s^2 + 4s + 8}$ ← Compensated CLTF

CLTF: $G(s) = \dfrac{1}{s(s+2)}$

$\Rightarrow \dfrac{1}{s(s+2) + 1}$

$= \dfrac{1}{s^2 + 2s + 1}$ ← uncompensated CLTF



Step Responses of Uncompensated and Compensated System
Outputs
t seconds
Uncompensated
Compensated
0
0.2
0.4
0.6
0.8
1
1.2
0.5
1.5
2
2.5
3
3.5
4
4.5
5

3)

Angle of difiency

$\tan^{-1}\left(\frac{2}{2\sqrt{3}}\right) = 30$

$\theta_1 = 30 + 90 = 120$

$\theta_2 = 90$

$\phi = -180 - \theta_{Total}$

$\phi = -180 - (-210) = 30$

$G_c^{ld}(s) = K \frac{s+4}{s+7}$

Characteristic

$1 = |K\, G_c(s)\, C(s)|$

$K = \left| \frac{s+7}{s+4} \cdot \frac{s(0.5s+1)}{5} \right|$

$s = -2 + 3.464j$

$K = \left| \frac{-15.603 - 35.76668j}{10 + 15.464j} \right| = \left| \frac{-15.603}{10} \right| = 1.56$

$G_c^{ld}(s) = K \frac{s+z}{s+p} \Rightarrow 1.56 \frac{s+4}{s+7}$

$K_c = 1.56$

$z = 4$

$p = 7$

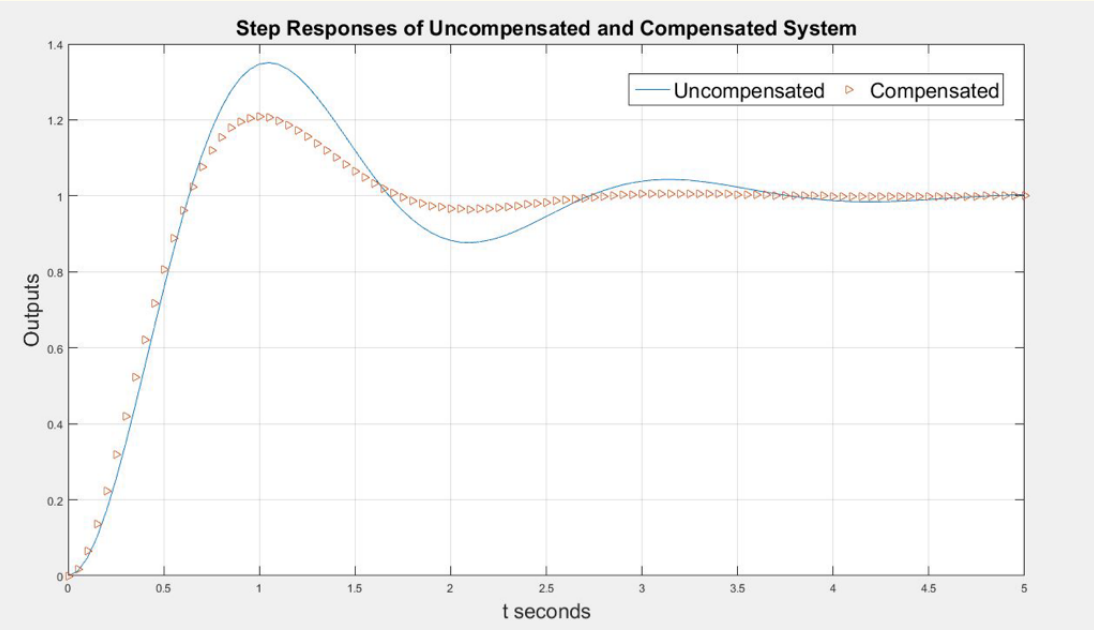
$G(s) = \frac{5}{s(0.5s+1)}$

CLTF: $\dfrac{1.56 \frac{s+4}{s+7} \frac{5}{s(0.5s+1)}}{1 + 1.56 \frac{s+4}{s+7} \frac{5}{s(0.5s+1)}}$

$= \frac{7.8s + 31.2}{0.5s^3 + 4.5s^2 + 14.8s + 31.2}$ ← Compensated

CLTF: $\frac{5}{0.5s^2 + s + 5}$

$= \frac{5}{0.5s^2 + s + 5}$ ← Uncompensated


Step Responses of Uncompensated and Compensated System
Uncompensated
Compensated
Outputs
t seconds
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0.5
1.5
2
2.5
3
3.5
4
4.5
5

4) assume:

$$1 + K(\ldots) = K(\ldots)$$

$$\Rightarrow \frac{U(s)}{E(s)} = \frac{K}{1 + K\left(\frac{1}{K_0} \cdot \frac{T_1 s}{1+T_1 s} \cdot \frac{1}{1+T_2 s}\right)}$$

$$= \frac{\cancel{K}}{\cancel{K}\left(\frac{1}{K_0} \cdot \frac{T_1 s}{1+T_1 s} \cdot \frac{1}{1+T_2 s}\right)}$$

$$= \frac{1}{\frac{1}{K_0} \cdot \frac{T_1 s}{1+T_1 s} \cdot \frac{1}{1+T_2 s}}$$

$$= \frac{K_0 (1+T_1 s)(1+T_2 s)}{T_1 s}$$

$$= K_0 \left(\frac{1}{T_1 s} + 1\right)(1 + T_2 s)$$

$$= K_0 \left(\frac{1}{T_1 s} + \frac{T_2 \cancel{s}}{T_1 \cancel{s}} + 1 + T_2 s\right)$$

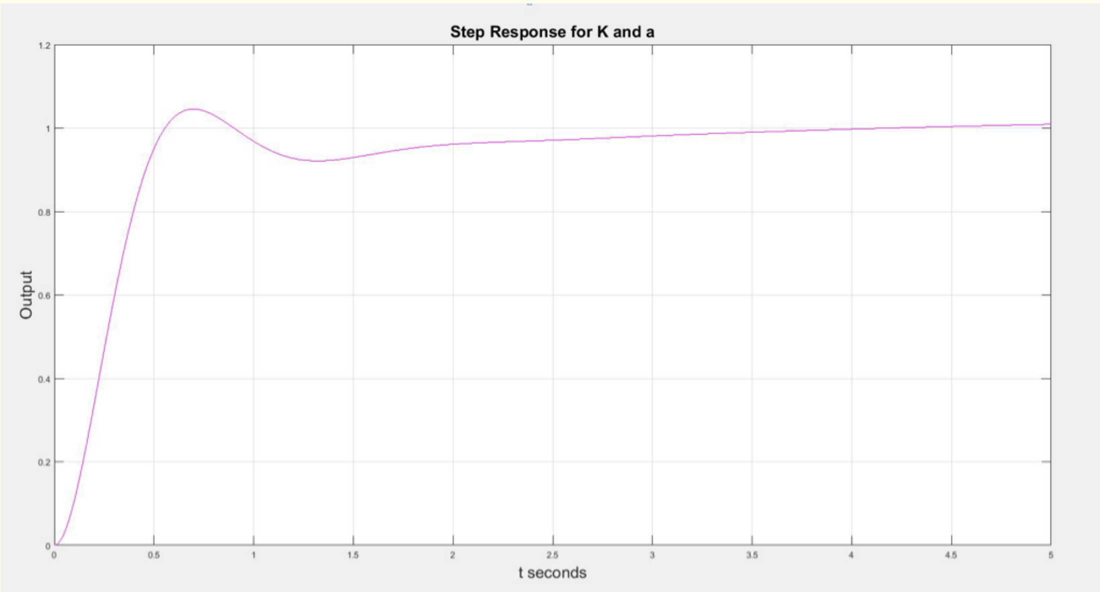
$$\underbrace{\frac{T_2 \cancel{s}}{T_1 \cancel{s}} + 1}_{\frac{T_2 + T_1}{T_1}}$$

$$= K_0 \frac{T_2 + T_1}{T_1}\left(1 + \frac{\cancel{T_1}}{T_2 + T_1} \cdot \frac{1}{\cancel{T_1} s} + \frac{T_1}{T_2 + T_1} \cdot T_2 s\right)$$

$$= K_0 \frac{T_2 + T_1}{T_1}\left(1 + \frac{1}{(T_2 + T_1)s} + \frac{T_1 T_2 s}{T_2 + T_1}\right)$$

5)

```
%Homework 8, problem 5
syms s
syms t
syms K
syms a

t = 0:0.01:5; %t in seconds for defining time
for K = 2:1:50; %K parameters
for a = 0.05:0.05:2; %a parameters
    num =[K 2*K*a K*a^2]; %numerator of the CLTF
    den =[1 6 5+K 2*K*a K*a^2]; %denominator of the CLTF
    Y = step(num,den,t); %plot of the CLTF
    X = stepinfo(Y); %list of properties
    O = X.Overshoot; %line used in list of properties
    s = 501; %found with command length(time)
        while Y(s)>0.98 && Y(s)<1.02; %finding the settling time at 2% critieria

        s = s-1;
        end;
    ts = (s-1)*0.01;

if (O)<10 & (O)>2 && ts<3.0 % settling time and overshoot conditions must be met for a
    break;

end
end
if (O)<10 & (O>2) && ts<3.0 % settling time and overshoot conditions must be met for K
   break
end
end
%solutions
plot(t,Y)
xlabel('t seconds')
ylabel('Output')
title('Step Response for K and a')
Optimal_K_and_a = [K;a]
Max_Overshoot_and_Settling_Time = [(O);ts]
```


Step Response for K and a
Output
t seconds
0
0.2
0.4
0.6
0.8
1
1.2
0.5
1.5
2
2.5
3
3.5
4
4.5
5

HOMEWORK # 8, PROBLEM 5 SOLUTION

Ahmad F. Taha
April 17, 2016

For a unity feedback system, with $G(s) = \dfrac{1}{s(s+1)(s+5)}$, we want to design a PID controller of this form:

$$G_{PID}(s) = K\frac{(s+a)^2}{s},$$

where:

- $2 \leq K \leq 50$ and $0.05 \leq a \leq 2$,
- the desired maximum overshoot is between 2 and 10 percent,
- the settling time is less than 3 seconds,
- Time simulation is for 5 seconds,
- You can iterate K and a with increments of 1 and 0.1 respectively.

In this problem, you will have to write a MATLAB program that finds the optimal value for K and a given the above ranges for K and a and the design specifications. First, you will have to obtain the CLTF in terms of K and a (in standard form). After that, you'll have to write MATLAB code with two for loops (one that iterates K and other that iterates a) and if conditions that compute settling time and maximum overshoot. The output of your program should be the optimal K and a, as well as a plot that shows the step response.

Here's a pseudo-code that you can follow:

```
time = 0:0.01:5; % defining time
for K = 2:1:50 % for loop for K
for a = 0.05:0.1:2; $ for loop for a
num = [......]; % Numerator for the CLTF in terms of K, a
den = [.......]; % Denominator for the CLTF in terms of K, a
y_step = step(num,den,time); % generating the step response for
% the specific K and a
.....  % Your code here
..... % Your code here
..... % Your code here
..... % Here, in these few lines of code (above), you have to write
% statements which includes testing for the design specs that
% are required from you. You can use the max MATLAB function, if-else statements,
% breaks, and while to do that.
end % ending the first for loop
end % ending the second for loop
plot(time,y_step)
ylabel('Output')
solution = .... % Extract your solution from the loops
```

Solutions (from Ogata):

The closed-loop transfer function $C(s)/R(s)$ is given by

$$\frac{C(s)}{R(s)} = \frac{Ks^2 + 2Kas + Ka^2}{s^4 + 6s^3 + (5 + K)s^2 + 2Kas + Ka^2}$$

A possible MATLAB program that will produce the first set of variables K and a that will satisfy the given specifications is given in MATLAB Program 8–15. In this program we use two 'for' loops. The specification for the settling time is interpreted by the following four lines:

$$\text{s} = 501; \text{while y(s)} > 0.98 \text{ and y(s)} < 1.02;$$
$$\text{s} = \text{s} - 1; \text{end};$$
$$\text{ts} = (\text{s} - 1) * 0.01$$
$$\text{ts} < 3.0$$

Note that for $t = 0{:}0.01{:}5$, we have 501 computing time points. $s = 501$ corresponds to the last computing time point.

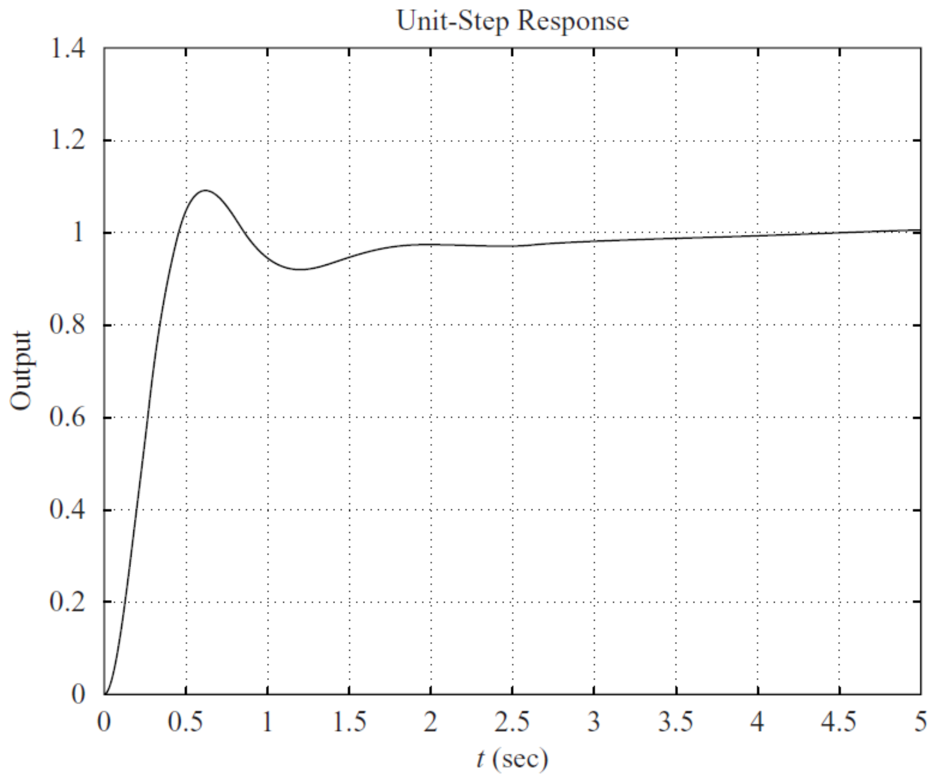
The solution obtained by this program is

$$K = 32, \qquad a = 0.2$$

with the maximum overshoot equal to 9.69% and the settling time equal to 2.64 sec. The resulting unit-step response curve is shown in Figure 8–64.

MATLAB Program 8–15

```
t = 0:0.01:5;
for K = 50:-1:2;
  for a = 2:-0.05:0.05;
    num = [K  2*K*a  K*a^2];
    den = [1  6  5+K  2*K*a  K*a^2];
      y = step(num,den,t);
      m = max(y);
      s = 501; while y(s) > 0.98 & y(s) < 1.02;
      s = s-1; end;
      ts = (s-1)*0.01;
    if m < 1.10 & m > 1.02 & ts < 3.0
    break;
    end
    end
  if m < 1.10 & m > 1.02 & ts < 3.0
  break
  end
  end
plot(t,y)
grid
title('Unit-Step Response')
xlabel('t sec')
ylabel('Output')
solution = [K;a;m;ts]

solution =

32.0000
  0.2000
  1.0969
  2.6400
```



Next, we shall consider the case where we want to find all sets of variables that will satisfy the given specifications. A possible MATLAB program for this purpose is given in MATLAB Program 8–16. Note that in the table shown in the program, the last row of the table (k, :) or the first row of the sorttable should be ignored. (These are the last *K* and *a* values for searching purposes.)

MATLAB Program 8–16

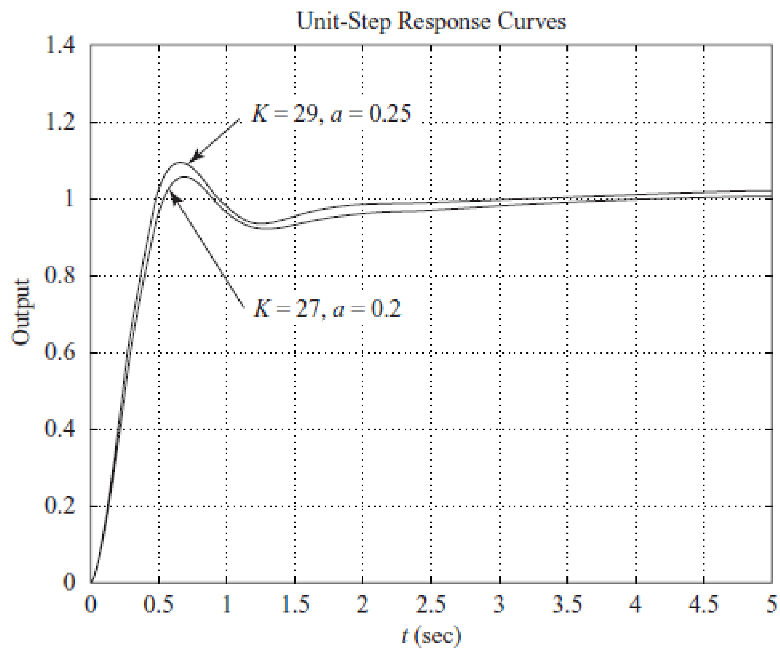
```
t = 0:0.01:5;
k = 0;
for i = 1:49;
   K(i) = 51-i*1;
      for j = 1:40;
      a(j) = 2.05-j*0.05;
      num = [K(i)  2*K(i)*a(j)  K(i)*a(j)*a(j)];
      den = [1  6  5+K(i)  2*K(i)*a(j)  K(i)*a(j)*a(j)];
         y = step(num,den,t);
         m = max(y);
         s = 501; while y(s) > 0.98 & y(s) < 1.02;
         s = s-1; end;
         ts = (s-1)*0.01;
      if m < 1.10 & m > 1.02 & ts < 3.0
      k = k+1;
      table(k,:) = [K(i)  a(j)  m  ts];
      end
   end
 end
table(k,:) = [K(i)  a(j)  m  ts]

table =
```

(*continues on next page*)

```
   32.0000  0.2000  1.0969  2.6400
   31.0000  0.2000  1.0890  2.6900
   30.0000  0.2000  1.0809  2.7300
   29.0000  0.2500  1.0952  1.7800
   29.0000  0.2000  1.0726  2.7800
   28.0000  0.2000  1.0639  2.8300
   27.0000  0.2000  1.0550  2.8900
    2.0000  0.0500  0.3781  5.0000

sorttable = sortrows(table,3)

sorttable =

    2.0000  0.0500  0.3781  5.0000
   27.0000  0.2000  1.0550  2.8900
   28.0000  0.2000  1.0639  2.8300
   29.0000  0.2000  1.0726  2.7800
   30.0000  0.2000  1.0809  2.7300
   31.0000  0.2000  1.0890  2.6900
   29.0000  0.2500  1.0952  1.7800
   32.0000  0.2000  1.0969  2.6400

K = sorttable(7,1)

K =

   29

a = sorttable(7,2)

a=

   0.2500

num = [K  2*K*a  K*a^2];
den = [1  6  5+K  2*K*a  K*a^2];
y = step(num,den,t);
plot(t,y)
grid
hold
Current plot held
K = sorttable(2,1)

K=

   27

a = sorttable(2,2)

a=

   0.2000
```

(*continues on next page*)

```
num = [K  2*K*a  K*a^2];
den = [1  6  5+K  2*K*a  K*a^2];
y = step(num,den,t);
plot(t,y)
title('Unit-Step Response Curves')
xlabel('t (sec)')
ylabel('Output')
text(1.22,1.22,'K = 29, a = 0.25')
text(1.22,0.72,'K = 27, a = 0.2')
```



From the sorttable, it seems that

$$K = 29, a = 0.25 \text{ (max overshoot } = 9.52\%, \text{settling time } = 1.78 \text{ sec)}$$

and

$$K = 27, a = 0.2 \text{ (max overshoot } = 5.5\%, \text{settling time } = 2.89 \text{ sec)}$$

are two of the best choices. The unit-step response curves for these two cases are shown in Figure 8–65. From these curves, we might conclude that the best choice depends on the system objective. If a small maximum overshoot is desired, $K = 27, a = 0.2$ will be the best choice. If the shorter settling time is more important than a small maximum overshoot, then $K = 29, a = 0.25$ will be the best choice.