

Module 08

Controller Designs: Compensators and PIDs

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EE 3413: Analysis and Design of Control Systems

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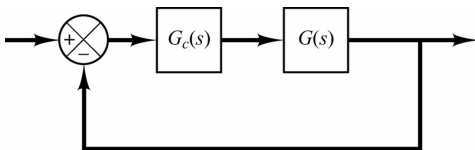
Webpage: <http://engineering.utsa.edu/ataha>



March 19, 2020

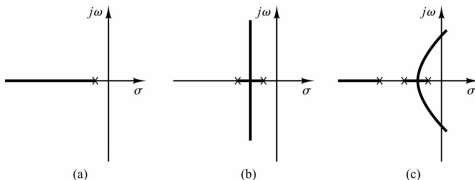
Introduction

- Readings: 6.5–6.6, 8.1–8.2 Ogata; 7.6,10.3,10.5 Dorf & Bishop
- In Module 7, we learned to sketch the RL for any TF
- We saw how poles change as a function of the gain K
- 'K' was a controller — a constant controller
- Many times, K as a controller is not enough
- Example: system cannot be stabilized with a choice of K -gain
- Or, settling time is still high, overshoot still bad
- Today, we'll learn how to design more complicated controllers
- **Objective:** find $G_c(s)$ such that CLTF has desired properties such as settling time, maximum overshoot,...

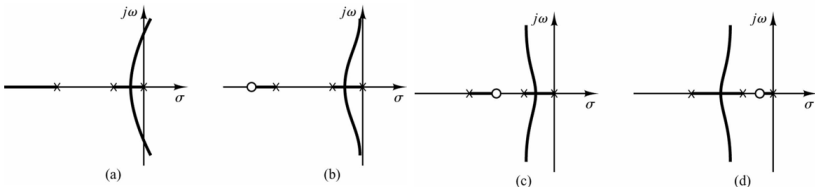


Typical RL Plots

- (a) Root-locus plot of a single-pole system; (b) root-locus plot of a two-pole system; (c) root-locus plot of a three-pole system



- (a) Root-locus plot of a three-pole system; (b), (c), and (d) root-locus plots showing effects of addition of a zero to the three-pole system.

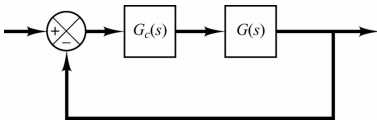


Effects of Adding Poles and Zeros on RL

- Adding poles *pulls* the RL to the **right**
 - Systems become “less stable”, settling is slower
- Adding zeros *pulls* the RL to the **left**
 - Systems become “more stable” (this is tricky), settling is faster
- **Question:** Can we conclude that a compensator (controller) $G_c(s)$ should always be a combination of zeros? Since, you know, it makes system *more stable* and settling is faster?
- Not really. **Why??** Because adding a zero amplifies the *high frequency noise*
- So, we can't add a zero alone (i.e., $G_c(s) = s + z$), and we can't add a pole alone either ($G_c(s) = \frac{1}{s+p}$). Solution?
- **Solution — Add a compensator of this form:**

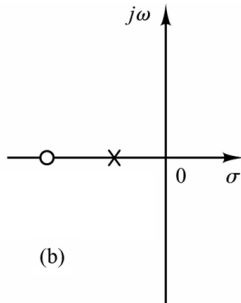
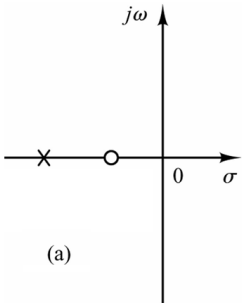
$$G_c(s) = K \frac{s + z}{s + p} \text{ — Objective: find } K, z, p \text{ given certain desired properties}$$

Two Controller Choices: Lead and Lag Compensators

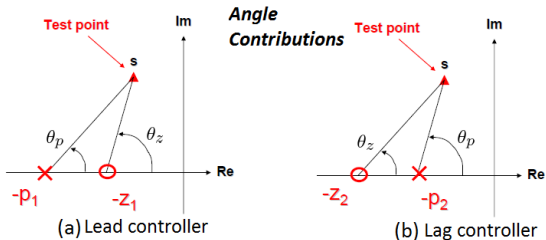


$G_c(s) = K \frac{s + z}{s + p}$ — Objective: find K, z, p given certain desired properties

- For $G_c(s)$ above, K, z, p are all **real +ve** values to be found
- \Rightarrow 2 combinations: (a) **lead controller**; (b) **lag controller**:



Lead and Lag Compensators



- **Lead compensator** provides a +ve angle contribution:

$$G_c^{ld}(s) = K \frac{s+z}{s+p} \Rightarrow \angle G_c^{ld}(s) = \angle(s+z) - \angle(s+p) = \theta_z - \theta_p = \theta_{lead} > 0$$

- Speeds up transients by lowering rise time & decreasing overshoots

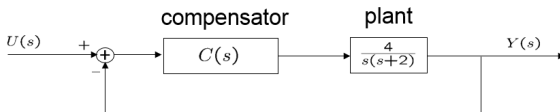
- **Lag compensator** provides a -ve angle contribution:

$$G_c^{ld}(s) = K \frac{s+z}{s+p} \Rightarrow \angle G_c^{ld}(s) = \angle(s+z) - \angle(s+p) = \theta_z - \theta_p = \theta_{lag} < 0$$

- Improves the steady-state accuracy of the system for tracking inputs

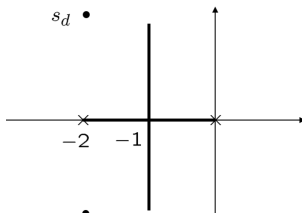
- What if $p = z$? That's a constant gain (pole & zero cancel out)

Lead Compensator Example



- Initially, the above system has $\zeta = 0.5$ and $\omega_n = 2$
- Obj:** design $G_c^{ld}(s) = C(s) = K \frac{s+z}{s+p}$, such that $\zeta_d = 0.5, \omega_{nd} = 4$
- Can we do that via gain K ? **No, see the RL below for $C(s) = K$**
- Hence, we can **never** reach s_d via a constant gain, need compensator

$$s_d = -\zeta_d \omega_{nd} \pm \sqrt{1 - \zeta_d^2} \omega_{nd} = -2 \pm j2\sqrt{3}$$



Lead Compensator Example (Cont'd)

- **Objective:** design $G_c^{ld}(s) = K \frac{s+z}{s+p}$, such that $\zeta_d = 0.5, \omega_{nd} = 4$

- To find K, z, p , follow this algorithm:

0. Find s_d for $s_d^2 + 2\zeta_d\omega_{nd}s_d + \omega_{nd}^2 = 0, s_d = -2 + j2\sqrt{3}$

1. Find *angle of deficiency* ϕ , as follows:

$$\theta = \angle G(s_d) = \angle G(-2 + j2\sqrt{3}) = -210 \text{ deg} \Rightarrow \phi = -180 - (\theta) = 30 \text{ deg}$$

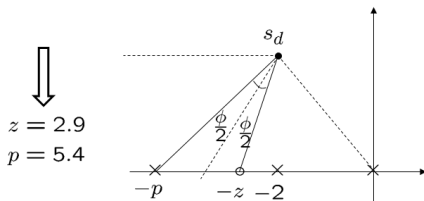
2. Connect s_d to the origin — OK

3. Draw a horizontal line to the left from s_d — OK

4. Find the bisector of the above two lines — OK

5. Draw 2 lines that make angles $\phi/2$ & $-\phi/2$ with the bisector — OK

6. Their intersections with the real lines are $-p$ and $-z$ — OK



Lead Compensator Example — Finding K

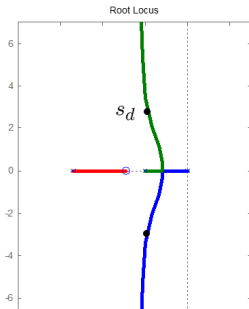
- We now know that $z = 2.9, p = 5.4 \Rightarrow G_c^{ld}(s) = C(s) = K \frac{s+2.9}{s+5.4}$
- We know that all points on the RL satisfy

$$1 + KG(s)G_c^{ld}(s) = 0 \Rightarrow \boxed{1 = |KG(s)G_c^{ld}(s)|} \quad \text{here } G_c^{ld}(s) \text{ is without the } K$$

- We know **for sure** that s_d belongs to the RL, so solve for K :

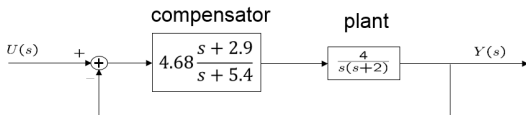
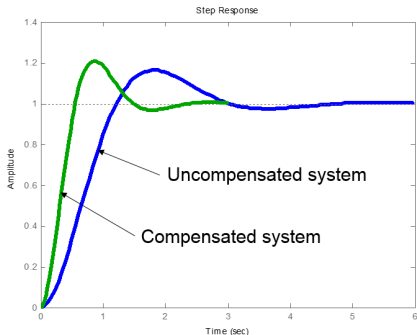
$$\left| 4K \frac{s_d + 2.9}{s_d(s_d + 2)(s_d + 5.4)} \right| = 1 \Rightarrow K = 4.68 \Rightarrow G_c^{ld}(s) = C(s) = 4.68 \frac{s + 2.9}{s + 5.4}$$

- *Compensated* RL plot:



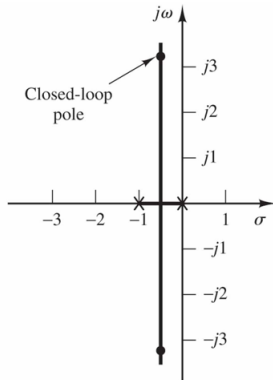
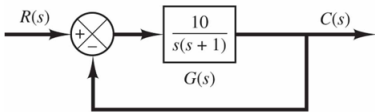
Step Response: Old vs. New Response

Compensated system reaches SS faster (shorter rise, settling times), although it has a higher M_p . That said, we designed the compensator according to the design specs. Design specs weren't so smart, perhaps.



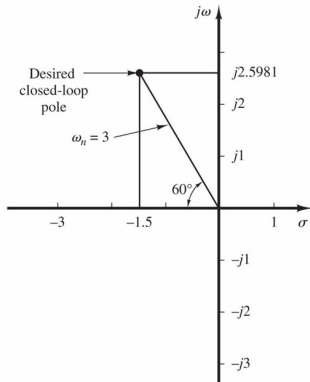
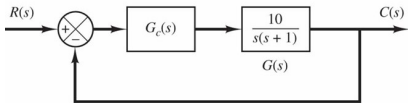
Lead Compensator Example 2

- Given $G(s) = \frac{10}{s(s+1)}$, find $G_c^{ld}(s)$ such that the CLTF has $\zeta_d = 0.5$ and $\omega_{nd} = 3$
- Figures: (a) uncompensated control system (b) uncompensated root-locus plot



Lead Compensator Example 2 (Cont'd)

- Given $G(s) = \frac{10}{s(s+1)}$, find $G_c^{ld}(s)$ such that the CLTF has $\zeta_d = 0.5$ and $\omega_{nd} = 3$
- Figures: (a) compensated system, (b) desired closed-loop pole location



Lead Compensator Example 2 (Cont'd)

● **Objective:** design $G_c^{ld}(s) = K \frac{s+z}{s+p}$, such that $\zeta_d = 0.5, \omega_{nd} = 3$

0. Find s_d for $s_d^2 + 2\zeta_d\omega_{nd}s_d + \omega_{nd}^2 = 0, s_d = -1.5 \pm j2.58$

1. Find *angle of deficiency* ϕ , as follows:

$$\theta = \angle G(s_d) = \angle G(-1.5 + j2.58) = 138 \text{ deg} \Rightarrow \phi = -180 - (138) = -318 \equiv 42 \text{ deg}$$

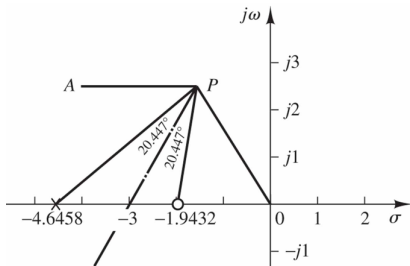
2. Connect s_d to the origin — OK

3. Draw a horizontal line to the left from s_d — OK

4. Find the bisector of the above two lines — OK

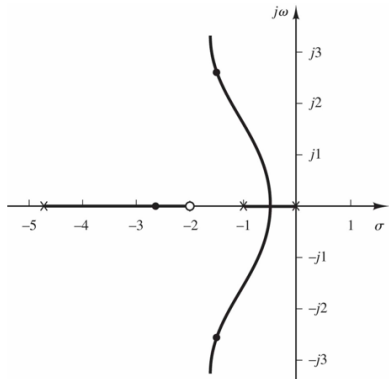
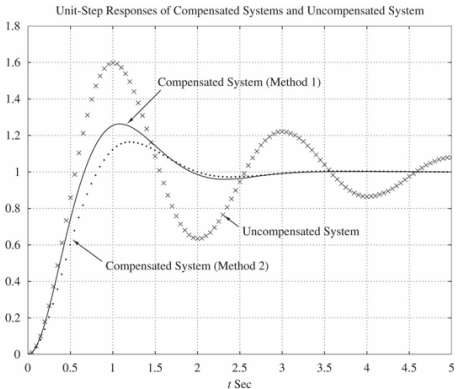
5. Draw 2 lines that make angles $\phi/2$ & $-\phi/2$ with the bisector — OK

6. Their intersections with the real lines are $-p$ and $-z$ — OK



Compensated System, Example 2

Unit-step response and RL plot for the compensated system:

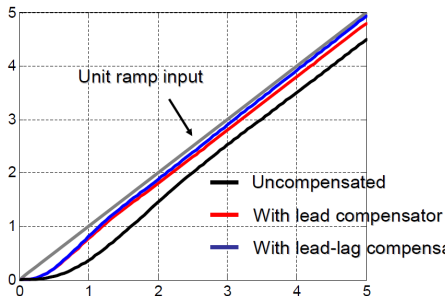
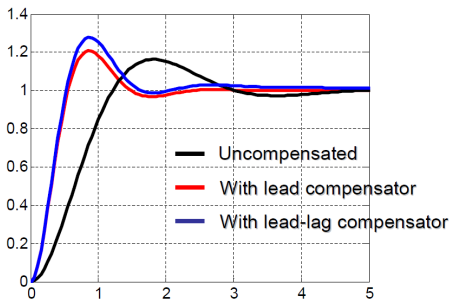


Lag, Lead-Lag Compensators

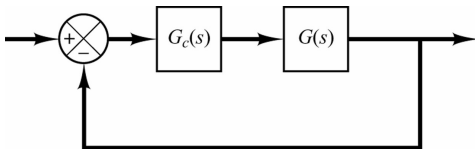
- Now that we understand lead compensators, we can discuss lag and lead-lag compensators
- Recall that lead compensators: improve transient response and stability
- But they do not typically reduce SSE
- Lag compensators $G_c^{lg}(s)$: reduce SSE, so sometimes we want smaller SSE rather than shorter rise and settling time as in a lead compensator
- **Optimal solution:** lead-lag (LL) compensator—
$$G_c^{ll}(s) = G_c^{ld}(s)G_c^{lg}(s)$$
- LL compensators provides the benefits of both lead and lag compensators

Lead-Lag Compensators

- Unfortunately, we don't have time to cover LL compensator design
- Design procedure is simple, please read more about it from your textbooks
- But we'll show a figure that illustrates the difference in performance:
- Left figure (step response), right figure (ramp response)



PID Control — Definitions and Basics



- **P**roportional **I**ntegral **D**erivative controller—PID control
- Without a doubt the most widely used controllers in industry today
- Bread and butter of control, 90% of control loops use PID control
- Proportional: $G_c(s) = K$, Integration: $G_c(s) = \frac{1}{T_s}$, Derivative: $G_c(s) = Ks$
- Can have combinations of the above controllers: P,I,D,PI,PD,ID,PID
- Major objectives for designing $G_c(s)$:
 - 1 Stability—the most important objective: CLTF is stable
 - 2 Steady-state error (SSE)—minimize this as much as we could
 - 3 Time-specs— M_p, t_r, t_s, \dots

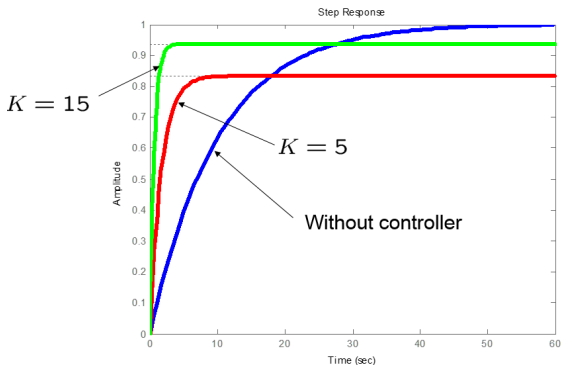
PID Controller Device



You can either use some tuning rules (which we will learn about during this module), or use an auto-tune function that figures out the parameters to a PID controller. Check <http://www.omega.com/prodinfo/temperaturecontrollers.html> for examples. Prices for common PID controllers range from \$20 to \$200, depending on size, quality, and performance.

Example 1: P controller for FOS

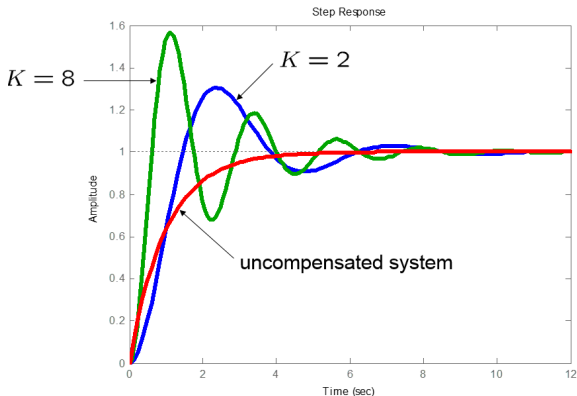
- Assume $G(s) = \frac{1}{Ts+1}$ —first order system (FOS)
- We can design a P controller (i.e., $G_c(s) = K$)



- Result:
- Larger K will increase the response speed
- SSE is present no matter how large K is—recall the SSE Table ;)

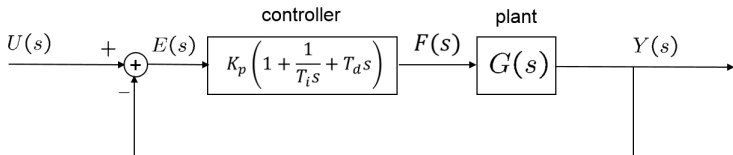
Example 2: Integral (I) controller for FOS

- Assume $G(s) = \frac{1}{Ts+1}$ —first order system (FOS)
- We can design an I controller (i.e., $G_c(s) = K/s$)



- Result:
- SSE for step input is completely eliminated
- But transients are bad—can cause instability for some K

PID Controller

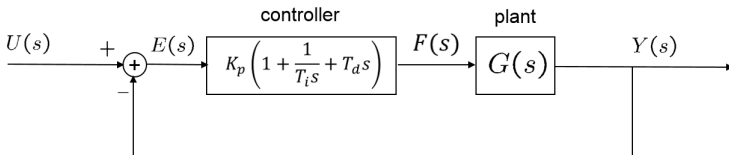


- PID (Proportional-Integral-Derivative) controller takes this form:

$$G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right)$$

- Design objective: find parameters K_p , T_i , T_d given required specs
- This process is called **PID tuning**—process of adjusting K_p , T_i , T_d
- Many different **tuning rules** exist
- **Ziegler-Nichols Rule**: first PID tuning rules (first and second method)
- After finding these parameters, input them on the PID device

More on PID Controllers



- Proportional term, i.e., $G_c^P(s) = K_p$:
 - Proportional term responds immediately to the current tracking error. Typically, however, it cannot achieve the desired tracking accuracy without excessively large gain.
- Integral term, i.e., $G_c^I(s) = \frac{K_p}{T_i s}$:
 - Integral term yields a zero steady-state error in tracking a step function (a constant set-point). This term is slow in response to the current tracking error.
- Derivative term, i.e., $G_c^D(s) = K_p T_d s$:
 - Derivative term is effective for plants with large dead-time
 - Reduces transient overshoots, but amplifies higher frequencies sensor noise

Ziegler-Nichols Rule: First Method



Step 1 Obtain plant's unit step response experimentally¹

- Unit step response is S-shaped for many plants
- Only valid if the step-response is S-shaped

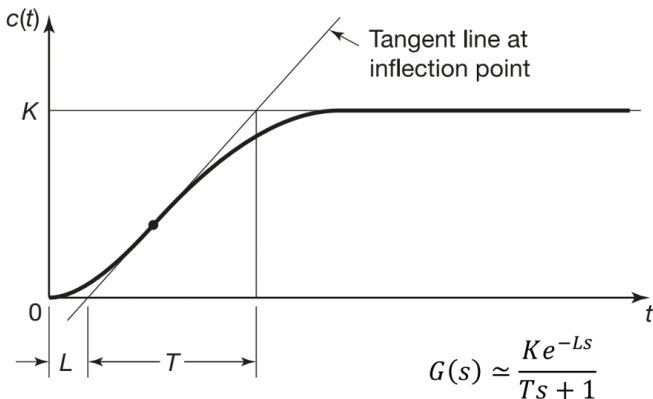
Step 2 Obtain delay time L from the experimental plot

Step 3 Obtain time constant T from the experimental plot

Step 4 Use tuning rule table to determine K_p, T_i, T_d given L, T (next slide)

¹In industrial applications, control engineers usually specify the performance of the controlled system based on the system step response.

Obtaining L , T from Experimental Plot



- Of course, this is an approximation, but you have to be accurate with your computation of L and T

Obtaining K_p , T_i , T_d via Tuning Method 1

$$G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right)$$

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{KL}$	∞	0
PI	$\frac{0.9T}{KL}$	$3.3L$	0
PID	$1.2 \frac{T}{KL}$	$2L$	$0.5L$

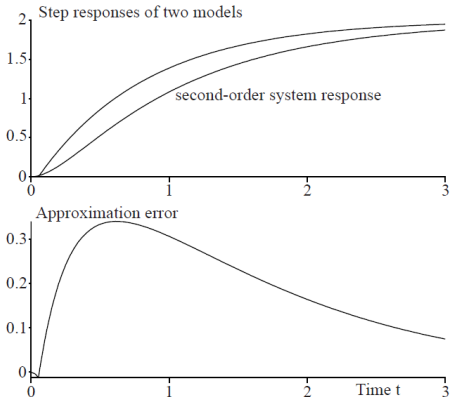
- If you want PID control, choose the 3rd row & compute the parameters:

$$G_{PID}(s) = G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right) = 0.6T \frac{\left(s + \frac{1}{L}\right)^2}{s}$$

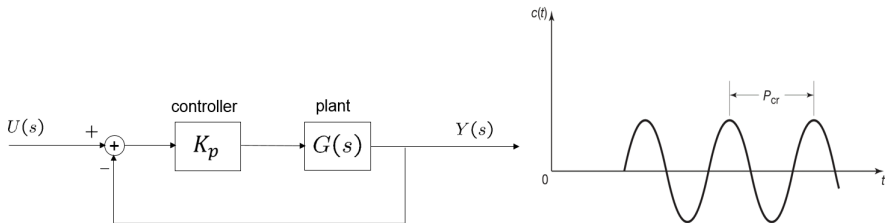
- These rules give only the starting point for the design—you might get a better response if you obtain a different set of constants

PID Tuning, Method 1 Example

- Given a plant $G(s) = \frac{10}{s^2 + 6s + 5}$, find K, L, T first
- Given the procedure, we find that $\tilde{G}(s) \approx \frac{2e^{-0.05s}}{0.8s + 1}$
- Plug these values in the table to obtain $G_c(s) = G_{PID}(s)$



Ziegler-Nichols Rule: Second Method



Step 1 Set $T_i = \infty$, $T_d = 0$ (above left figure) and increase K_p until step response of the closed-loop system **has sustained oscillations**

- If no oscillation occurs for all values of K_p , this method is not applicable

Step 2 Record K_{cr} (critical value of gain K_p) and P_{cr} (period of the oscillation); see above right figure

Step 3 Use tuning rule table to determine K_p , T_i , T_d given K_{cr} , P_{cr} (next slide)

Obtaining K_p , T_i , T_d via Tuning Method 2

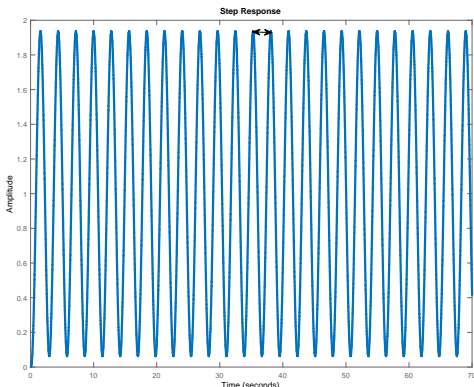
$$G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right)$$

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$P_{cr}/1.2$	0
PID	$0.6K_{cr}$	$P_{cr}/2$	$P_{cr}/8$

$$G_{PID}(s) = G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right) = 0.075K_{cr}P_{cr} \frac{\left(s + \frac{4}{P_{cr}} \right)^2}{s}$$

Method 2 Example

- Given a plant $G(s) = \frac{1}{s(s+1)(s+5)}$, find the PID parameters using the second PID design method
- Solution:** Experimentally, we plot the step response till we have sustained oscillations (Step 1)
- We can record $K_{cr} = 30$, $P_{cr} = 2.8$



Method 2 Example

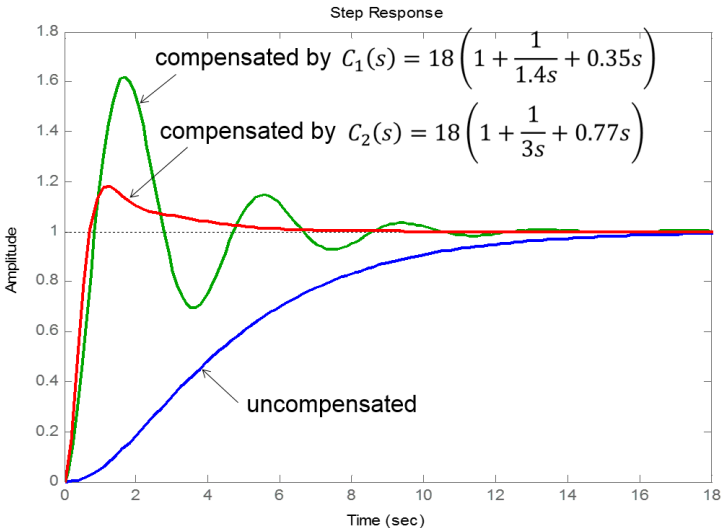
- Looking at the table, we can find K_p, T_i, T_d :

$$G_c(s) = 18 \left(1 + \frac{1}{1.4s} + 0.35s \right)$$

- Note:** we can find K_{cr} by applying the RH table for the CP ($s^3 + 6s^2 + 5s + K_p$)
- Then, you can find K_p that would make the CP unstable
 $\Rightarrow K_{cr} = K_p^{max} = 30$
- Then find the frequency ω_{cr} that solves this equation

$$(j\omega_{cr})^3 + 6(j\omega_{cr})^2 + 5j\omega_{cr} + 30 = 0 \Rightarrow \omega_{cr} \Rightarrow \omega_{cr} = \sqrt{5} \Rightarrow P_{cr} = \frac{2\pi}{\omega_{cr}} = 2.8$$

Step Response after PID Design



PID Control Summary

- We only covered one type of PID control, called Type A PID control:

$$G_{PID-A}(s) = G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right) = \alpha \frac{(s + \beta)^2}{s}$$

where α and β are the PID constants that depend on the plant's performance

- So, when do we use P,PI,PD, or PID control?
- Well, it depends on what you want

↗ Parameter	SSE	Response Speed	Stability	Oscillations	Overshoot
↗ K_p	↘	↗	↘	↗	↗
↗ $K_i = \frac{1}{T_i}$	↘	↘	↘	↗	↗
↗ $K_d = T_d$	↗	↗	↗	↘	↘

Course Progress

Modeling
(5-6 Weeks)

- Laplace Transforms
- Transfer Functions
- Solution of ODEs
- Modeling of Systems
- Block Diagrams
- Linearization

Analysis
(7-8 Weeks)

- 1st & 2nd Order Systems
- Time Response
- Transient & Steady State
- Frequency Response
- **Bode Plots**
- RH Criterion
- Stability Analysis

Design
(5-6 Weeks)

- Root-Locus
- Design via RL, Compensators
- PID Control
- Modern Control
- State-Space
- MIMO System Properties

Questions And Suggestions?



Thank You!

Please visit

engineering.utsa.edu/ataha

IFF you want to know more 😊