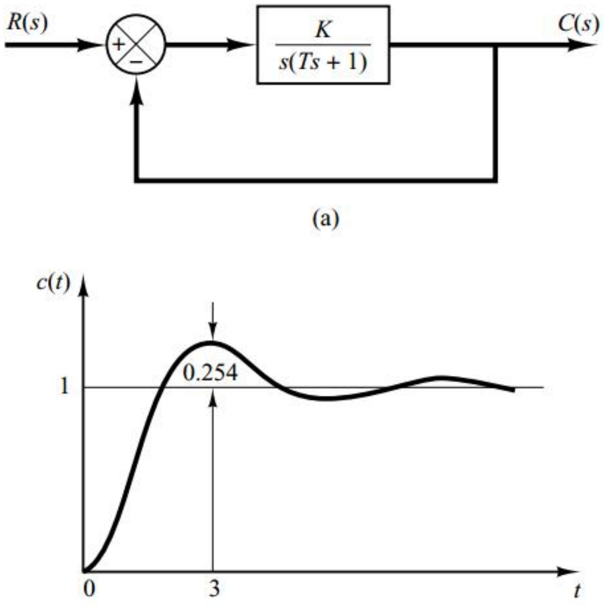
A–5–3. When the system shown in Figure 5–52(a) is subjected to a unit-step input, the system output responds as shown in Figure 5–52(b). Determine the values of K and T from the response curve.

Solution. The maximum overshoot of 25.4% corresponds to $\zeta = 0.4$. From the response curve we have

$$t_p = 3$$

Consequently,

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega_n\sqrt{1-0.4^2}} = 3$$



(a)

(b)

Figure 5–52
(a) Closed-loop system; (b) unit-step response curve.

It follows that

$$\omega_n = 1.14$$

From the block diagram we have

$$\frac{C(s)}{R(s)} = \frac{K}{Ts^2 + s + K}$$

from which

$$\omega_n = \sqrt{\frac{K}{T}}, \qquad 2\zeta\omega_n = \frac{1}{T}$$

Therefore, the values of T and K are determined as

$$T = \frac{1}{2\zeta\omega_n} = \frac{1}{2 \times 0.4 \times 1.14} = 1.09$$

$$K = \omega_n^2 T = 1.14^2 \times 1.09 = 1.42$$

A–5–4. Determine the values of K and k of the closed-loop system shown in Figure 5–53 so that the maximum overshoot in unit-step response is 25% and the peak time is 2 sec. Assume that $J = 1$ kg-m^2.

Solution. The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + Kks + K}$$

By substituting $J = 1$ kg-m^2 into this last equation, we have

$$\frac{C(s)}{R(s)} = \frac{K}{s^2 + Kks + K}$$

Note that in this problem

$$\omega_n = \sqrt{K}, \qquad 2\zeta\omega_n = Kk$$

The maximum overshoot M_p is

$$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

which is specified as 25%. Hence

$$e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.25$$

from which

$$\frac{\zeta\pi}{\sqrt{1-\zeta^2}} = 1.386$$

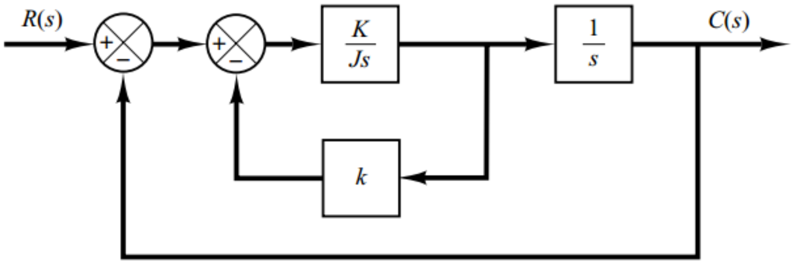


Figure 5–53
Closed-loop system.

or

$$\zeta = 0.404$$

The peak time t_p is specified as 2 sec. And so

$$t_p = \frac{\pi}{\omega_d} = 2$$

or

$$\omega_d = 1.57$$

Then the undamped natural frequency ω_n is

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{1.57}{\sqrt{1-0.404^2}} = 1.72$$

Therefore, we obtain

$$K = \omega_n^2 = 1.72^2 = 2.95 \text{ N-m}$$

$$k = \frac{2\zeta\omega_n}{K} = \frac{2 \times 0.404 \times 1.72}{2.95} = 0.471 \text{ sec}$$

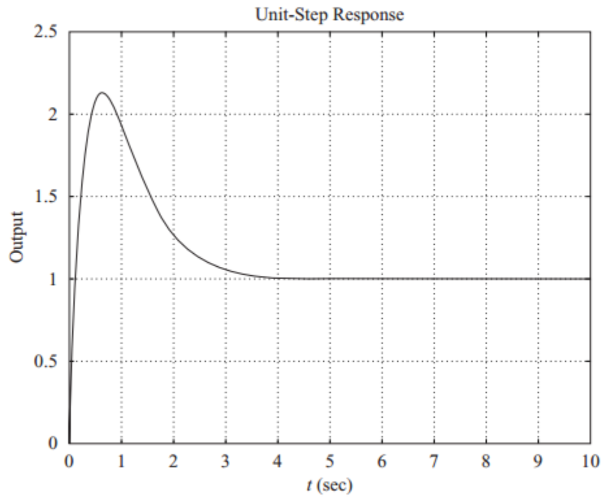
A–5–9. When the closed-loop system involves a numerator dynamics, the unit-step response curve may exhibit a large overshoot. Obtain the unit-step response of the following system with MATLAB:

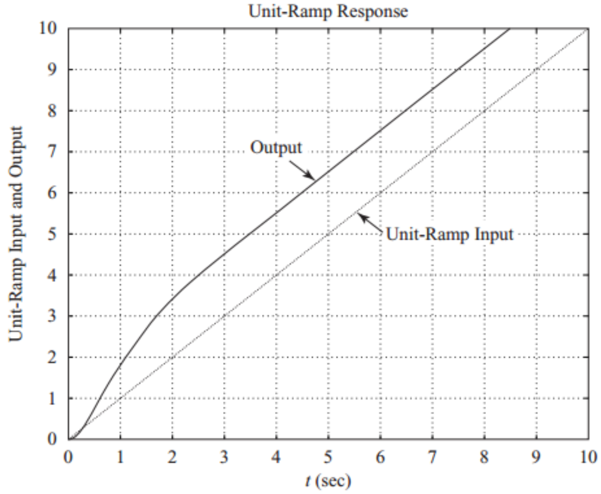
$$\frac{C(s)}{R(s)} = \frac{10s + 4}{s^2 + 4s + 4}$$

Obtain also the unit-ramp response with MATLAB.

Solution. MATLAB Program 5–19 produces the unit-step response as well as the unit-ramp response of the system. The unit-step response curve and unit-ramp response curve, together with the unit-ramp input, are shown in Figures 5–57(a) and (b), respectively.

Notice that the unit-step response curve exhibits over 215% of overshoot. The unit-ramp response curve leads the input curve. These phenomena occurred because of the presence of a large derivative term in the numerator.





MATLAB Program 5–19

```
num = [10  4];
den = [1  4  4];
t = 0:0.02:10;
y = step(num,den,t);
plot(t,y)
grid
title('Unit-Step Response')
xlabel('t (sec)')
ylabel('Output')

num1 = [10  4];
den1 = [1  4  4  0];
y1 = step(num1,den1,t);
plot(t,t,'--',t,y1)
v = [0  10  0  10]; axis(v);
grid
title('Unit-Ramp Response')
xlabel('t (sec)')
ylabel('Unit-Ramp Input and Output')
text(6.1,5.0,'Unit-Ramp Input')
text(3.5,7.1,'Output')
```

A–5–17. Consider the following characteristic equation:

$$s^4 + Ks^3 + s^2 + s + 1 = 0$$

Determine the range of K for stability.

Solution. The Routh array of coefficients is

$$\begin{array}{cccc} s^4 & 1 & 1 & 1 \\ s^3 & K & 1 & 0 \\ s^2 & \dfrac{K-1}{K} & 1 & \\ s^1 & 1-\dfrac{K^2}{K-1} & & \\ s^0 & 1 & & \end{array}$$

For stability, we require that

$$K > 0$$

$$\frac{K-1}{K} > 0$$

$$1 - \frac{K^2}{K-1} > 0$$

From the first and second conditions, K must be greater than 1. For $K > 1$, notice that the term $1 - \left[K^2/(K-1)\right]$ is always negative, since

$$\frac{K - 1 - K^2}{K - 1} = \frac{-1 + K(1 - K)}{K - 1} < 0$$

Thus, the three conditions cannot be fulfilled simultaneously. Therefore, there is no value of K that allows stability of the system.