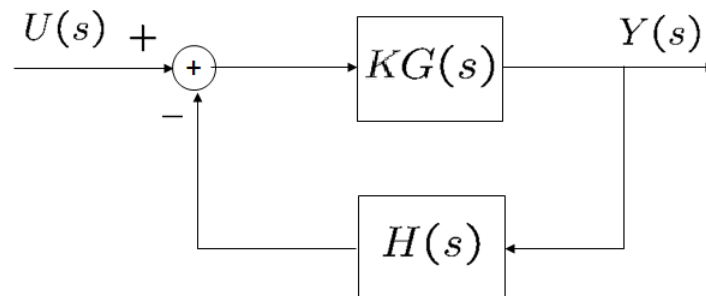


Root-Locus: 10 Simple Rules

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March 19, 2020



Defs. OLTF = $G(s)H(s)$; n_p, n_z = number of poles, zeros of OLTF; Characteristic Polynomial (CP) = $1 + KG(s)H(s)$

$$\Rightarrow 1 + KG(s)H(s) = 0 \Rightarrow K(s) = \frac{-1}{G(s)H(s)}$$

Rule 1 RL is always symmetric with respect to **the real-axis**—remember that

Rule 2 RL has n branches, $n = n_p$

Rule 3 Mark poles (n_p) and zeros (n_z) of $G(s)H(s)$ with 'x' and 'o'

Rule 4 Each branch starts at OLTF poles ($K = 0$), ends at OLTF zeros or at infinity ($K = \infty$)

Rule 5 RL has branches on x-axis. These branches exist on real axis portions where the **total # of poles + zeros** to the right is an odd #

Rule 6 Asymptotes angles: RL branches ending at OL zeros at ∞ approach the asymptotic lines with angles:

$$\phi_q = \frac{(1 + 2q)180}{n_p - n_z} \text{ deg}, \forall q = 0, 1, 2, \dots, n_p - n_z - 1$$

Rule 7 Real-axis intercept of asymptotes:

$$\sigma_A = \frac{\sum_{i=1}^{n_p} \text{Re}(p_i) - \sum_{j=1}^{n_z} \text{Re}(z_j)}{n_p - n_z}$$

Rule 8-1 RL branches intersect the real-axis at points where K is at an extremum for real values of s . Remember that:

$$1 + KG(s)H(s) = 0 \Rightarrow K(s) = \frac{-1}{G(s)H(s)}$$

We find the breakaway points by finding solutions (i.e., s^* solutions) to:

$$\frac{dK(s)}{ds} = 0 = -\frac{d}{ds} \left[\frac{1}{G(s)H(s)} \right] = 0 \Rightarrow \frac{d}{ds} [G(s)H(s)] = 0 \Rightarrow \text{obtain } s^*$$

Rule 8-2 After finding s^* solutions (you can have a few), check whether the corresponding

$$K(s^*) = \frac{-1}{G(s^*)H(s^*)} = K^* \text{ is } \mathbf{\text{real positive \#}}$$

Rule 8-3 Breakaway pt.: K_{max}^* (-ve $K''(s^*)$), **Break-in pt:** K_{min}^* (+ve $K''(s^*)$)

Rule 9 Angle of Departure (AoD): defined as the angle from a complex pole or Angle of Arrival (AoA) at a complex zero:

$$\text{AoD from a complex pole : } \phi_p = 180 - \sum_i \angle p_i + \sum_j \angle z_j,$$

$$\text{AoA at a complex zero : } \phi_z = 180 + \sum_i \angle p_i - \sum_j \angle z_j$$

– $\sum_i \angle p_i$ is the sum of all angles of vectors to a complex pole in question from other poles, $\sum_j \angle z_j$ is the sum of all angles of vectors to a complex pole in question from other zeros

– ‘ \angle ’ denotes the angle of a complex number

Rule 10 Determine whether the RL crosses the imaginary y-axis by setting:

$$1 + KG(s = j\omega)H(s = j\omega) = 0 + 0i$$

and finding the ω and K that solves the above equation. The value of ω you get is the frequency at which the RL crosses the imaginary y-axis and the K you get is the associated gain for the controller. You should obtain two equations (real = 0 and imaginary = 0) with two unknowns (K, ω). From there, you solve for K, ω pairs