Your Name: Your Signature:

- **Exam duration:** 1 hour and 30 minutes.
- This exam is closed book, closed notes, closed laptops, closed phones, closed tablets, closed pretty much everything.
- No bathroom break allowed.
- **If we find that a laptop, phone, tablet or any electronic device near or on a person and even if the electronics device is switched off, it will lead to a straight zero in the finals.**
- **No calculators** of any kind are allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, **even if your answer is correct.**
- Place a box around your final answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- This exam has 15 pages, plus this cover sheet. Please make sure that your exam is complete, that you read all the exam directions and rules.

Question Number	Maximum Points	Your Score
1	35	
2	20	
3	25	
4	20	
Total	100	

1. (35 total points) You are given the following LTI dynamical system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) \end{aligned}$$

where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

(a) (5 points) What are the eigenvalues of the system? And Is the system stable?

(b) (5 points) Is the above system controllable or not? Justify your answer.

(c) (5 points) What is the controllability subspace?

(d) (5 points) Is the above system observable or not? Justify your answer.

(e) (5 points) Obtain the unobservable subspace of the system.

(f) (5 points) Is there a state feedback controller $u(t) = -Kx(t)$ such that $A - BK$ has eigenvalues $\{-2, -2, -2\}$? If yes, find this state feedback gain K. Justify why if your answer is no.

(g) (5 points) Is there a state observer such that $A - LC$ has eigenvalues $\{1, -1, -2\}$? If yes, find this state feedback gain L. Justify why if your answer is no.

2. (20 total points) The following LTV system is given:

$$\dot{x}(t) = \begin{bmatrix} -\dfrac{1}{1+e^t} & \dfrac{1}{1+e^t} & 0 \\ 0 & \dfrac{-2}{1+e^t} & 0 \\ 0 & 0 & 10 \end{bmatrix} x(t).$$

Hint:

$$\int_{t_0}^{t} \frac{1}{1+e^t} dt = \int_{t_0}^{t} \frac{1+e^t-e^t}{1+e^t} dt = \int_{t_0}^{t} \left(\frac{1+e^t}{1+e^t} - \frac{e^t}{1+e^t} \right) dt = ...?$$

(a) (20 points) Find the state transition matrix of this system.

3. (25 total points) You are given the following SISO system:

$$\dot{x}(t) = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 4 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0.5 & 1 \end{bmatrix} x(t).$$

   (a) (20 points) Design an observer based controller (i.e., $u(t) = -K\hat{x}(t)$) for the above system such that the desired eigenvalues for the closed loop system are all at $\lambda_{cl} = -5$ (i.e., for both $A - LC$ and $A - BK$).

   First, you'll have to check if the system is controllable and observable (or detectable and stabilizable).

(b) (5 points) Draw a block diagram representation of the overall system with the observer based controller, including the values for the gains K and L that you have designed.

4. (20 total points) Consider the following nonlinear system:

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)x_2(t) - 2x_1(t) \\ \dot{x}_2(t) &= x_1(t) - x_2(t) - 1. \end{aligned}$$

   (a) (10 points) Find all the equilibrium points of the nonlinear system. You should find at least 2 equilibrium points.

(b) (10 points) Determine the stability of the system around each equilibrium point.