Name:

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1. You are given this system:

$$x(k+1) = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix} x(k) + \begin{bmatrix} 2 \\ 2 \end{bmatrix} u(k), \quad u(k) = 0.5^k, \quad x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

   Find $x(n)$. You might find this equation to be useful:

$$\boxed{x(k) = A^k x(0) + \sum_{j=0}^{k-1} A^{k-1-j} B u(j) = A^k x(0) + \sum_{j=0}^{k-1} A^j B u(k-1-j)}$$

2. Consider this dynamical system

$$x(k+1) = A(k)x(k) + B(k)u(k), y(k) = C(k)x(k) + Du(k).$$

Note that A, B, C are all **time-varying**. Given that you have four sets of input-output data:

$$(y(0), u(0)), (y(1), u(1)), (y(2), u(3)), (y(3), u(3))$$

and $x(0)$ is unknown, derive an equation that would allow you to obtain $x(0)$ as follows:

$$\bar{A}x(0) = \bar{b}$$

where $\bar{A}$ and $\bar{b}$ are quantities that you should determine, then discuss the conditions for $\bar{A}$ that allows you to generate a unique $x(0)$. Note that in this problem, the number of states is much larger than the output measurements.